Electric Power Systems Research 209 (2022) 108046

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Sizing and Allocation of DGs in A Passive Distribution Network Under Various Loading Scenarios

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ARTICLE INFO

Keywords:
Analytical approach
passive distribution network
optimization
different characteristic generators
ZIP load model

ABSTRACT

Integration of renewable energies in distribution system networks has grown significantly over the years. Finding the optimal location and precise capacity of distributed generations (DGs) in various load conditions is a profound challenge. This article proposes an analytical approach to find the optimal location and capacity of different characteristic DGs in a passive distribution network (PDN). Real-life scenario of power consumption is considered by using different load scenarios. Key benefits include loss reduction and improved voltage profile of the PDN. Special attention is paid for the role of DGs to provide active/reactive power in different load scenarios. Monetary benefits are calculated for optimized installed generators. Extensive case studies have been performed over IEEE-33 and 69 bus system to verify the benefits of the proposed methodology.

1. Introduction

1.1. Background and Motivation

Operation and control of a power system depends on the distribution system which is the linkage between the high voltage transmission system and end consumers. A passive distribution network (PDN) generally consists of main feeder and lateral distributors. The power flows from the substation to end consumer in a root-to-leaf structure. The electrical power transferred from generating station to power consumers through transmission and distribution systems is accompanied by losses. The larger part of these losses takes place in distribution networks. The distribution losses are about 50% of total losses in a power system. So much attention is required for the reduction of losses in distribution systems.

Integration of distributed generations (DGs) has proved as one of the most efficient approaches for loss reduction and improving power quality in distribution systems. Optimal sizing and appropriate location selection plays a key role in the best utilization of DGs. An unutilized or overstressed DG causes monetary losses and creates operational issues in the system. Appropriate DG integration not only enables loss reduction but also provides monetary benefit in operational activity. Renewable energy resources mostly coupled with power-electronic inverter/converter interface are capable of improving the power quality. Distributed generators provide active and reactive power both as per the end consumer demands. These DGs can be classified into three different categories based on their characteristic of power supply.

1. Type-I DG injects only active power.
2. Type-II DG injects only reactive power.
3. Type-III DG injects both active and reactive powers.

Also, real-life loads respond differently with the variation of voltage and frequency. It is necessary to analyze V-I characteristics of loads to perform an efficient load flow. Various researchers considered a constant active and reactive model, but this model is insufficient to consider real-life loads. For ease of computation different load models are expressed as active and reactive power with the function of voltage and frequency.

1.2. Literature Review

The optimal planning of distributed energy resources are necessary in a power system [1]. Various researchers refer to different techniques in literature for finding the optimal location and capacity for the installation of DGs into the distribution system. The optimization techniques such as heuristic, analytical, and hybrid are investigated for this purpose. One of the heuristic approaches is particle swarm optimization (PSO) for finding the optimal location and size of DGs [2]. The optimal

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https://doi.org/10.1016/j.epsr.2022.108046
Received 24 March 2021; Received in revised form 4 April 2022; Accepted 24 April 2022
Available online 28 April 2022

Nomenclature

DG	Distributed generation.
PDN	Passive distribution network.
P_L	Total active power loss of PDN.
P_i	Active power injection at i^{th} node.
Q_i	Reactive power injection at i^{th} node.
V_i	Nodal voltage magnitude at i^{th} node.
δ_i	Voltage angle at i^{th} node.
r_{mn}	Resistance of branch *mn*.
x_{mn}	reactance of branch *mn*.
I_{mn}	Branch current between node *m* and *n*.
$P_{(G,m)}$	Active power generations of generator at m^{th} node.
$P_{(D,m)}$	Active power demands at m^{th} node.
$Q_{(G,m)}$	Reactive power generations of generator at m^{th} node.
$Q_{(D,m)}$	Reactive power demands at m^{th} node.
G_{mn}	Conductance of branch *mn*.
B_{mn}	Susceptance of branch *mn*.
P_{inj}	Active power injection.
Q_{inj}	Reactive power injection.
CP	Constant power.
CI	Constant current.
CZ	Constant impedance.
ZIP	Constant impedance, current, and power.
P_G	Active power generation.
Q_G	Reactive power generation.
$C(P_G)$	Cost component of DG for active power in \$/(MWh).
$C(Q_G)$	Cost component of DG for reactive power in \$/(MWh).

solutions are provided to distribution utilities [2]. This technique is used for solving the capacitor allocation problem connected to wind energy generation. Due to the intermittent nature of wind energy, a non-linear fitness function has been involved [3]. The different types of DGs are considered in [4] for their optimal placement in the distribution system using PSO. Genetic algorithm (GA) is another heuristic approach for power loss reduction and voltage profile improvement using DG integration [5]. Genetic algorithm is also investigated for optimal allocation of synchronous condensers with improvement in short circuit ratio at a transmission system [6]. Prominently used heuristic approaches are computationally heavy and consume much more time than analytical approaches. The risk of getting a local optimal solution is higher in a heuristic approach and the computation time increases rapidly with the increase of system size.

An analytical approach is proposed for optimal placement and sizing of DG on distribution networks considering two novel bus types P and PQV buses [7]. Fuzzy logic is an intelligent technique to control the reactive power of DGs. The fuzzy system is optimized by a gradient descent algorithm and implemented on different types of DGs [8]. The combination of fuzzy system and GA develops a hybrid approach for optimization of DG parameters in deregulated power systems [9]. The impact of DG allocation with different load models is analyzed using a multi-objective shuffled bat algorithm [10]. The problem of multiple DG placements to obtain high loss reduction is presented in a large scale primary distribution network using mathematical expressions [11]. The voltage stability analysis considering the loop configuration of the distribution network is carried out, and voltage stability is evaluated in terms of a voltage stability index [12]. The hybridization of PSO and analytical technique is addressed in [13], for the optimal installation of multiple DGs.

Congestion management is a major problem in a large power system network, and it is solved by implementing a hybrid approach with optimal placement of DGs. The hybridization of the firefly technique and differential evaluation is developed [14]. The Shapley value method calculates the loss allocation sequentially for the radial distribution system. This method has main advantage i.e., it reduces the computational burden and memory requirement [15]. A meta-heuristic technique based on a backtracking search algorithm continuation power flow method is addressed for the modeling and stability analysis of the distribution system with high penetration of DGs [16],[17]. The improved continuation power flow method is executed for the modeling and stability analysis of the distribution system with high penetration of DGs. The distributed slack bus model based on incremental loss factors is employed to allocate the unbalanced power [18].

Distributed generators are integrated directly into a distribution network for delivering the power to local distribution consumers. But their integration will change network fault level requiring a proper protection scheme. A detailed review of protection schemes for bulk integration of renewable energy resources in a power system is presented in [19]. A multi-agent-based rolling optimization method for electrical distribution system restoration scheduling is presented giving an effective solution in a blackout event [20]. The voltage profile is improved via coordinated regulation of active and reactive power of DGs by using a randomized algorithm [21]. The distinct studies are carried out for solving the DG installation problem with load changes in [22–26]. Nowadays, power system planning engineers are much concerned to load changes while integrating the DGs in the distribution system and consider it an important area of research in the power system. Table 1 provides a comparative analysis of different techniques as the result of the literature survey.

1.3. Research Contribution

This article proposes an analytical method based optimized installation (sizing and location) of DGs. In literature, most of the researchers proposed DG installation using a constant power model of load, which is inadequate to consider all real-life scenarios. This article examined test-bed with various loading scenarios to verify the achievement of optimal solution by proposed algorithm. Reactive power suppliers/compensators also play important role in smooth operation of a PDN. Hence optimal sizing and location of type-II DG are also important.

The key contributions of the paper are as follows.

1. Modeling and optimization of different characteristic DGs for radial distribution system.
2. Computation of reduction in power loss, cost of energy loss, and improvement in voltage profile.
3. Impact of different loading scenarios on optimal installation of DGs.
4. Simpler mathematical expressions have been established for the optimization.

1.4. Paper Organization

The rest of the article is organized as follows. Section 2 provides the mathematical modeling of system components. It covers modeling of different characteristic DGs, and loads. It also describes the load flow method used for the analysis. Formulation of objective function for DG installation including network constraints and proposed analytical approach to solve the problem are explained in Section 3. Section 3 also describes the computational procedure for optimal placement of DGs, calculative steps for optimal power flow (OPF), annual cost of energy losses, and DG powers variation with load models. Test-bed description and result comparative analysis is done in Section 4. This section justifies the benefit of the analytical approach over existing literature

Table 1
Literature analysis.

Ref.	Optimal Placement	Optimal Sizing	Loss Reduction	Different Characteristic DG	Load Scenarios	Monetary Benefits	Technique
[2]	✓	✓	✓	✓	×	×	Heuristic (PSO)
[3]	✓	✓	✓	×	×	✓	Heuristic (PSO)
[4]	✓	✓	✓	✓	×	×	Heuristic (PSO)
[5]	✓	✓	✓	✓	✓	×	Heuristic (GA)
[7]	✓	✓	✓	✓	×	×	Analytical
[8]	×	×	✓	✓	×	×	Intelligent Technique (Fuzzy Logic)
[14]	✓	✓	✓	×	×	✓	Hybrid
Proposed	✓	✓	✓	✓	✓	✓	Analytical

methods. Finally, the paper is concluded in Section 5.

2. Mathematical Modeling of System Components

A Passive distribution network containing a distributed generation consists of three key components; generator, network, and load. These components need to model properly for accurate results and minimum error. This research paper focuses on modeling of various characteristic DGs and loads to cover whole real-life scenario. For load flow computation purposes tried and tested forward-backward approach is used.

2.1. DG Modeling

Based on DG characteristic, method of connection, and operation mode, DGs are classified as type-I, II, and III DG. These DG can be further classified as PQ or PV bus of radial distribution system. In this study only PQ characteristic DGs are selected, these DGs are classified as follows [27].

1. Type-I DG: DG injecting only active power (*P*) to the system, e.g. photovoltaic system, fuel cell, and battery (DG operating at unity power factor (PF)).
2. Type-II DG: DG injecting only reactive power (*Q*) to the system for improvement in voltage profile, e.g. capacitor and synchronous condenser (DG operating at zero PF).
3. Type-III DG: DG injecting both (*P*) and (*Q*) to the system, e.g. synchronous generator (DG operating at lagging PF).

The optimal sizes of these different characteristic DGs are determined in Section 3 of this article.

2.2. Load Modeling

A passive distribution network consist of various characteristic loads. It is inaccurate to model all these loads as constant PQ load. More accuracy can be achieved by modeling these loads as a function of voltage and frequency. The static load models are more relevant since these models are expressed as the steady state active power *(P)* and reactive power *(Q)* as a function of voltage and frequency. The *(P)* and *(Q)* values for these models are expressed as follows.

$$P = P_0 \left\{ \frac{v}{v_0} \right\}^{\gamma_p} \tag{1}$$

$$Q = Q_0 \left\{ \frac{v}{v_0} \right\}^{\gamma_q} \tag{2}$$

P_0 and Q_0 are active and reactive power components at nominal bus voltage v_0; γ_p and γ_q are load exponents; and v is nodal voltage. The system loads are modeled by assigning the specified values to load exponents. Therefore, based on specified exponent values the different load models are described in Table 2.

PDN contains static as well as combination of different static loads. Hence, it is important to model combination of static loads as ZIP load.

ZIP load model: It demonstrates realistic load model and consists of all above-mentioned load models i.e. CP, CI, and CZ. It characterizes the power and voltage relation as a polynomial function of the voltage. The *P* and *Q* characteristics of the load model are expressed as follows.

$$P = P_0 \left[\omega_{p1} \left(\frac{v}{v_0} \right)^2 + \omega_{p2} \left(\frac{v}{v_0} \right) + \omega_{p3} \right] \tag{3}$$

$$Q = Q_0 \left[\omega_{q1} \left(\frac{v}{v_0} \right)^2 + \omega_{q2} \left(\frac{v}{v_0} \right) + \omega_{q3} \right] \tag{4}$$

where sum of all coefficients of ZIP load model is equal to 1 for *P* and *Q* loads.

$$\omega_{p1} + \omega_{p2} + \omega_{p3} = 1 \tag{5}$$

$$\omega_{q1} + \omega_{q2} + \omega_{q3} = 1 \tag{6}$$

In this study, the ratio of different characteristic load combinations in ZIP load model is taken as follows.

$$\begin{aligned} \omega_{p1} &= \omega_{q1} = 10\% \ \ (CZ - Load) \\ \omega_{p2} &= \omega_{q2} = 10\% \ \ (CI - Load) \\ \omega_{p3} &= \omega_{q3} = 80\% \ \ (CP - Load) \end{aligned}$$

The base value of v_0 is considered as 1 p.u. This article visualizes the effect of CP, CI, CZ, and ZIP loads on optimal installation of different characteristic DGs.

2.3. Load Flow Method

The conventional methods of load flow such as Newton-Raphson and its modified versions, fast decoupled methods, etc. give better results in case of transmission systems but don't perform well in the distribution systems due to high R/X ratio. In this proposed work, tried and tested

Table 2
Load scenarios.

Load Models	Values of Load Exponents		Variation of P and Q
	γ_p	γ_q	
CP	0	0	Constant P and Q
CI	1	1	linear variation
CZ	2	2	Quadratic variation

forward-backward sweep load flow method [28]-[29] is used for efficient load flow of the passive distribution network. This method involves two-stage iterative process. In first stage, current is being calculated based on the nodal load in backward sweep[30]. In second stage, forward sweep updates the nodal voltage of all nodes. This iterative process ends when the voltage difference of two consecutive iterations reaches below the tolerance level.Figure 1 presents the line diagram and power injection at nodes (m) and (n). Where, (m) and (n) represents the sending and receiving end nodes for *(mn)* branch. The complex load S_{inj} of m^{th} node is given as.

$$S_{inj} = P_{inj} + jQ_{inj} \qquad \forall\ m\epsilon\eta \tag{7}$$

where η represents node buses of PDN. P_{inj} and Q_{inj} are active and reactive power injections at m^{th} node. Therefore, current calculation of m^{th} node for k^{th} iteration is done as.

$$I_m^k = \left(\frac{P_m + jQm}{V_m^k}\right)^* = I_m^r\left(V_m^k\right) + jI_k^r\left(V_m^k\right) \tag{8}$$

In backward sweep, the branch current is determined by the kirchhoff's current law using the nodal load current. The current updation equation is written as follows.

$$\overrightarrow{I}_{mn} = \overrightarrow{I}_{L_n} + \sum_{m\epsilon\eta} \overrightarrow{I}_{L_m} \tag{9}$$

Forward sweep is the next iterative step to update the voltage of each node in the system and is written below.

$$\overrightarrow{V}_n = \overrightarrow{V}_m - \overrightarrow{I}_{mn} Z_{mn} \tag{10}$$

Reason for selecting forward-backward approach is its efficiency, accuracy and wider acceptance for load flow solution of a PDN.

3. Problem Formulation

A generalized representation of problem formulation for analytical approach is defined as follows: minimize $\boldsymbol{P_L}$ = For optimized sizing and location of DGs. s.t. passive distribution network constraints.

In this study, a passive distribution network is assumed as a connected graph $\zeta = (\eta,\tau)$. Where η represents set of node buses of PDN and τ represents set of branch of PDN. Node 1 is connected with the substation (slack bus) of PDN. G and D represents the generation and demand of nodes.

3.1. Objective Function

The objective of this formulation is to minimize active power loss for optimized installation of DGs. Therefore, to find the active power loss in the PDN, Exact Loss formula described in (11) is used.

$$Minimize\ P_L = \sum_{m=1}^{\eta}\sum_{n=1}^{\eta}[\alpha_{mn}(P_mP_n + Q_mQ_n) + \beta_{mn}(Q_mP_n - P_mQ_n)] \tag{11}$$

Where,

$$\alpha_{mn} = \frac{r_{mn}}{V_mV_n}\cos(\delta_m - \delta_n) \tag{12}$$

$$\beta_{mn} = \frac{r_{mn}}{V_mV_n}\sin(\delta_m - \delta_n) \tag{13}$$

Exact loss formula uses power injection of adjacent node and line parameters to calculate total active power loss of network. Here, nodal complex power injection varies with the variation of the power injection of the DGs.

3.2. System Constraints

The connected graph ζ of a passive distribution network acts as a tree. Where $(m,n)\epsilon\tau$ denotes a linking branch between node m and n. Node $m\epsilon\eta$, where $m = 0, 1, 2..\eta$ represents PDN nodes with voltage and complex power injection. A passive distribution network has three key constraints: power, voltage, and current.

3.2.1. Power balance constraints

Each node should satisfy the power balance equations of the system, for both active and reactive power [14].

$$P_{(G,m)} - P_{(D,m)} = \sum_{n=1}^{\eta} V_mV_n[G_{mn}\cos(\delta_m - \delta_n) + B_{mn}\sin(\delta_m - \delta_n)] \qquad \forall(m,n)\epsilon\zeta \tag{14}$$

$$Q_{(G,m)} - Q_{(D,m)} = \sum_{n=1}^{\eta} V_mV_n[G_{mn}\sin(\delta_m - \delta_n) - B_{mn}\cos(\delta_m - \delta_n)] \qquad \forall(m,n)\epsilon\zeta \tag{15}$$

Nodal active and reactive power injections are calculated as follows.

$$P_{inj} = P_m = P_{(G,m)} - P_{(D,m)} \tag{16}$$

$$Q_{inj} = Q_m = Q_{(G,m)} - Q_{(D,m)} \tag{17}$$

Each feasible solution of optimal DG sizing and location must fulfill the power balance equation of load flow solution.

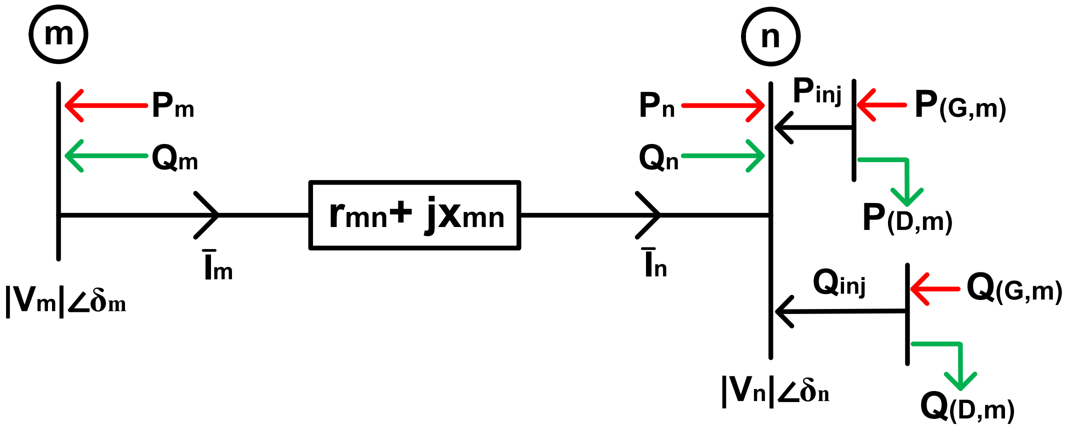

Fig. 1. Equivalent line diagram for PDN between node m and n.

3.2.2. *Voltage constraints*

Nodal voltage variations should be within the tolerance limit. In this study, ±10% of the rated voltage is considered as tolerance limit.

$$V_{min} \le V_m \le V_{max} \qquad \forall m \epsilon \eta \tag{18}$$

V_{max} and V_{min} are the upper and lower limits of voltage for all PDN nodes.

3.2.3. *Current constraints*

Due to the thermal limitation of conductor, PDN has limited current carrying capability. All branch currents should be within the permissible limit. Where, *pms* represents permissible limit and I_{mn}^{pms} is the maximum permissible current in branch *mn*.

$$I_{mn} \le I_{mn}^{pms} \qquad \forall (m,n) \epsilon \tau \tag{19}$$

Voltage and current limits for all the nodes are explained by Figure 2.

3.3. *Proposed Methodology*

Proposed methodology uses exact loss formulation to calculate active power loss in the PDN. Partial differentiation of network active power loss with respect to nodal active power injection gives the minima of power loss for optimal power injection. Partial differential equation and calculated optimal power injection are expressed in (20), and (21).

$$\frac{\partial P_L}{\partial P_m} = 2\alpha_{mm}P_m + 2\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}P_n - \beta_{mn}Q_n) = 0 \tag{20}$$

Equation (21), and (24) calculates the active and reactive power injection at node *m* for minimal power loss in PDN.

$$P_{inj} = -\frac{1}{\alpha_{mm}}\left[\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}P_n - \beta_{mn}Q_n)\right] \tag{21}$$

Equation (22), calculates the total active power injection by Type-I DG, and (25) calculates the total reactive power injection by Type-II DG at node *m*.

$$P_{(G,m)} = P_{(D,m)} - \frac{1}{\alpha_{mm}}\left[\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}P_n - \beta_{mn}Q_n)\right] \tag{22}$$

Similar to (20), partial derivative of network total active power loss with respect to nodal reactive power injection gives the optimal reactive power injection for network minimal active power loss. Equation is expressed in (23).

$$\frac{\partial P_L}{\partial Q_m} = 2\alpha_{mm}Q_m + 2\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}Q_n + \beta_{mn}P_n) = 0 \tag{23}$$

$$Q_{inj} = -\frac{1}{\alpha_{mm}}\left[\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}Q_n + \beta_{mn}P_n)\right] \tag{24}$$

$$Q_{(G,m)} = Q_{(D,m)} - \frac{1}{\alpha_{mm}}\left[\sum_{\substack{n=1\\ n\neq m}}^{\eta}(\alpha_{mn}Q_n + \beta_{mn}P_n)\right] \tag{25}$$

These equations calculate the optimal location and sizing of DGs along with fulfilling all constraints of the PDN for installation of Type-I and Type-II DG.

If the optimal location of Type-I and Type-II DG is on the same node *m*, then active power injection calculated by (22) and reactive power injection calculated by (25) needs to be supplied by Type-III DG at node *m*. Any size of DG other than $P_{(G,m)}$ and $Q_{(G,m)}$ installed at node *m* will cause higher active power loss. The nodal power factor (NOPF) of Type-III DG at node *m* is described as.

$$NOPF = \frac{P_{(G,m)}}{\sqrt{P_{(G,m)}^2 + Q_{(G,m)}^2}} \tag{26}$$

3.4. *Monetary Benefit Analysis*

This section focuses on the monetary benefit post optimization. The monetary benefits are occurring due to reduction in cost of energy loss as well as reduction in cost of powers obtained from DGs. Benefit analysis has been done on both IEEE-33 and 69 bus systems.

3.4.1. *Cost of energy losses*

The annual cost of energy losses (CEL) is given by [31].

$$CEL = (Total\,Active\,Power\,Loss) \times \left(K_p + K_e \times Lsf \times 8760\right)\$ \tag{27}$$

Here, the number 8760 reflects the total number of samples/hours for 365 days or equal to one year. Yearly calculation with hourly sampling rate 24*365 = 8760.

K_p : annual demand cost of power loss ($/kW)
K_e : annual cost of energy loss ($/kWh)
Lsf : loss factor
Loss factor is expressed in terms of load factor (Lf) as below.

$$Lsf = k \times Lf + (1-k) \times Lf^2 \tag{28}$$

The coefficient values used for (27) and (28) are given below.

$k = 0.2,\ Lf = 0.47,\ Kp = 57.6923\ \$/kW\ and\ Ke = 0.00961538\ \$/(kWh)$

3.4.2. *Cost of DGs active and reactive power*

The cost of active power supply for DGs are calculated using standard quadratic cost function.

$$C(P_G) = \left[a \times P_G^2 + b \times P_G + c\right]\frac{\$}{MWh} \tag{29}$$

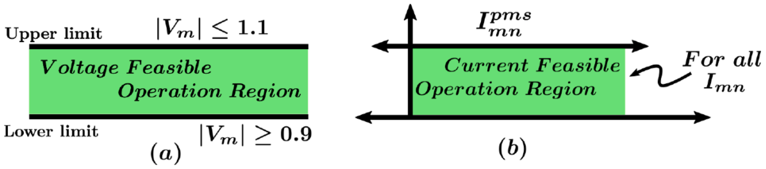

Fig. 2. (a) Nodal voltage and (b) branch current constraint for PDN.

The cost coefficients used for above calculation are as follows. $a = 0$, $b = 20$, $c = 0.25$

Similarly, the cost component of DGs reactive power is calculated using cost of complex power delivered and cost of active power delivered. The DGs reactive power cost component is expressed as follows.

$$C(Q_G) = \left[Cost\left(S_{(G,max)}\right) - Cost\left(\sqrt{S^2_{(G,max)} - Q^2_G}\right)\right] \times K \frac{\$}{MWh} \tag{30}$$

where,

$$S_{(G,max)} = \frac{P_{(G,max)}}{\cos\delta} \tag{31}$$

$$P_{(G,max)} = 1.1 \times P_G \tag{32}$$

where, $S_{(G,max)}$ and $P_{(G,max)}$ represents the maximum complex and active power generation limit. In this study, the constant K is assumed as 0.1.

Computational Procedure: The computational procedure to determine the optimal sizing and location of DGs under different loading scenarios and monetary benefit analysis for optimized DG installation is described as follows.

Step 1: Run load flow for PDN (without DG).
Step 2: Calculate the loss using (11).
Step 3: Compute the sizes of different characteristic DGs at each bus using (22) and (25) for minimum power loss.
Step 4: Check the violation of system constraints.
Step 5: Find the bus for minimum power loss.
Step 6: Compute NOPF using (26) in case of Type-III DG.
Step 7: Run load flow again with optimal DG size installed at optimal bus.
Step 8: Calculate active power loss reduction and voltage magnitude improvement with DG.
Step 9: Compute the monetary benefits using (27) to (32).
Step 10: Repeat all the above steps for different load scenarios.

4. Result and Discussion

Proposed methodology is tested on two standard IEEE-33 and 69 test-beds. For broader experimentation, proposed technique is implemented with different characteristic DGs under various load scenarios, e.g. CP, CI, CZ, and ZIP. Observation part consists of improvements in voltage profile, PDN loss reduction, and monetary benefit analysis. Based on DGs classification, results are produced with Type-I, II, III and simultaneously placed Type-I and II DGs. Simultaneously placed Type-I and II DGs can further be denoted as Type-IV DG. $(S)_{Base}$ and $(V)_{Base}$ are selected as 100 MVA and 12.66 kV for the investigation. Single line diagram for 33-bus system with different DG integrations are presented in Figure 3. A comparative analysis of study with literature findings are also presented in Section 4.3.

IEEE-33 bus system and IEEE-69 bus system data are available in [32]. All the simulations are performed with the help of Matlab version R2020b. Software code is generated using MATLAB environment for the implementation of proposed methodology.

4.1. IEEE-33 Test-bed

IEEE-33 test-bed is investigated in this section. Optimal sizing and location are investigated using PDN active loss calculation. Figure 4 presents the voltage profile improvement due to different characteristic DG integration. For different characteristic DGs integration, separate loss calculations are executed to obtain optimal DG power injection. Based on the minimal power losses optimal sizing and location are selected. Figure 5 represents the PDN active power loss due to various characteristic DG integration. For explanation purpose, in 33 bus system investigation, minimum loss and DG power injection graph for Type-III DG and ZIP load is presented in same Figure 6. It is understandable the reason to select Node-6 (since minima is occurring at node 6) for DG integration. Similar technique is used to evaluate location and sizing in each loading scenario.

All the numerical data calculated during this investigation e.g. optimal sizing, network power loss, and monetary analysis for various loading scenarios are presented in Table 3.

4.2. IEEE-69 Test-bed

In this section, the proposed methodology is tested on IEEE-69 test-bed. The objective of this test is to verify the applicability of proposed system over wider range of a passive distribution network. Similar, results are produced in this section. Figure 7 presents the voltage profile due to various characteristic DG integration on IEEE-69 bus system. Total active power loss for 69 node PDN under various DGs power injection is presented in Figure 8. Using the optimal nodal power injection and total power loss again optimal location is calculated. This calculation is done for ZIP load scenario and optimal location achieved is node-61, represented in Figure 9. Similar procedure is being followed for calculation of optimal sizing and location under CP, CI, and CZ loading conditions.

Numerical calculation is done to present the benefits of DGs integration and compare the values under different loading scenario. Monetary values, network loss and optimal sizing of DGs for various loading

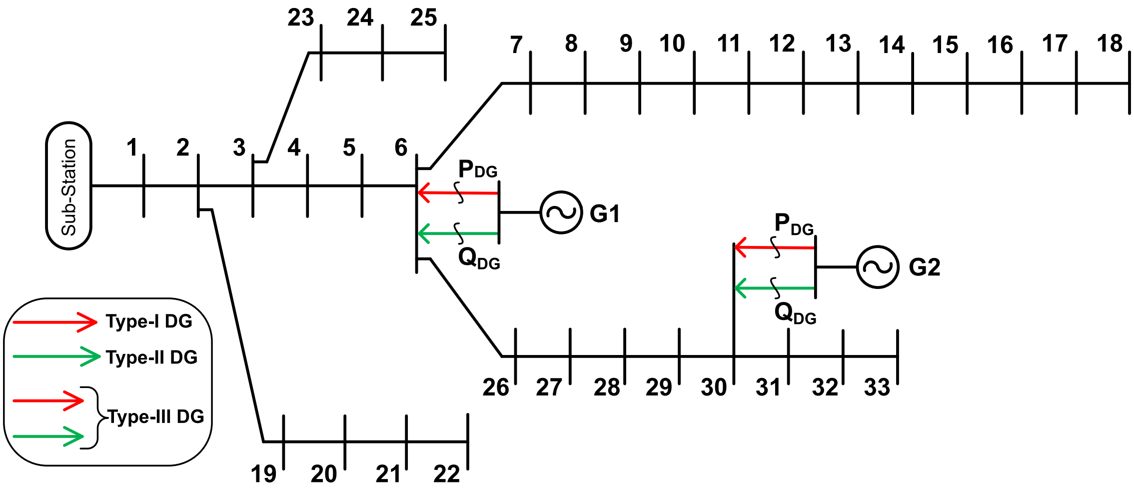

Fig. 3. Modified 33 bus passive distribution network with generators location.

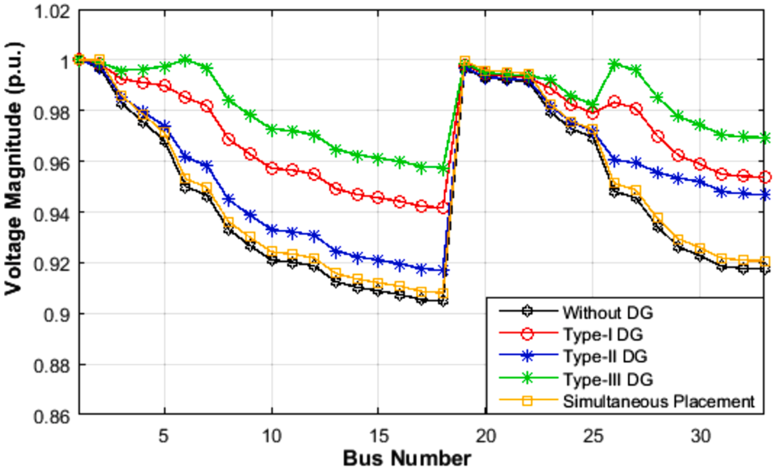

Fig. 4. Voltage profile of various DG installations for IEEE-33 test-bed.

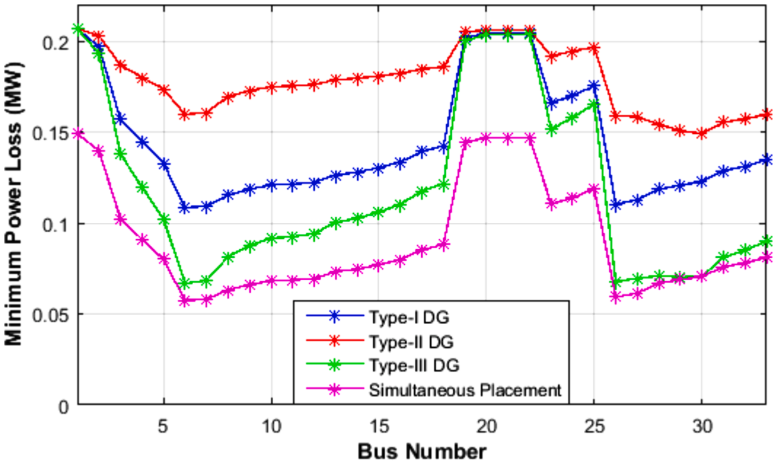

Fig. 5. Total power loss for IEEE-33 test-bed.

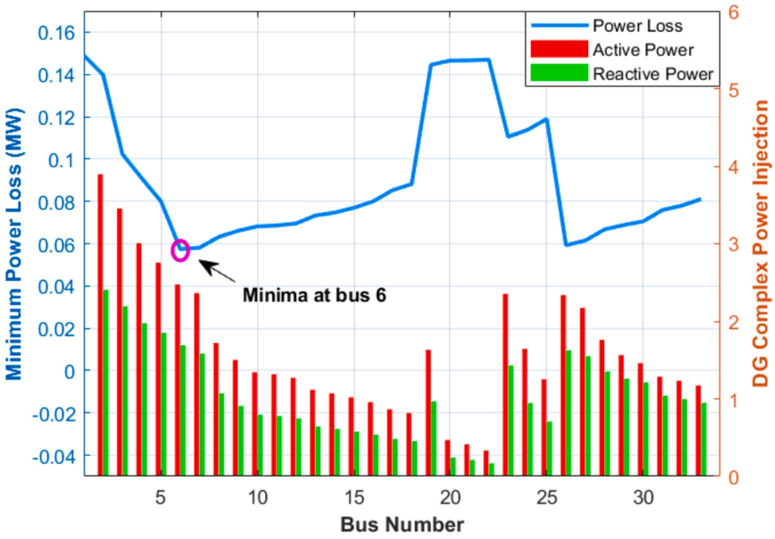

Fig. 6. Optimal location for IEEE-33 test-bed under ZIP load scenario.

Table 3
Optimal sizing and monetary benefit analysis for IEEE-33 test-bed.

Load	DG	Optimal DG Size			Active	Cost of Energy	P_G Cost	Q_G Cost
Scenario	Type	kW	kVAR	kVA (NOPF)	Power Loss (kW)	Losses ($)	($/MWh)	($/MVARh)
CP–Load	Type-I	2491	–	–	111.1	8935	50.1	–
	Type-II	–	1230	–	151.0	12155	–	15.8
	Type-III	–	–	3028 (0.82)	68.0	5474	50.1	9.6
	Simultaneous Placement	2491	1230	–	58.0	4669	50.1	5.2
CI–Load	Type-I	2320	–	–	94.0	7567	46.6	–
	Type-II	–	1130	–	130.0	10464	–	14.5
	Type-III	–	–	2807 (0.82)	58.0	4669	46.6	8.7
	Simultaneous Placement	2320	1130	–	50.0	4025	46.6	4.7
CZ–Load	Type-I	2140	–	–	84.0	6762	43.0	–
	Type-II	–	1060	–	112.0	9015	–	13.6
	Type-III	–	–	2613 (0.82)	51.0	4105	43.0	8.4
	Simultaneous Placement	2140	1060	–	45.0	3622	43.0	4.5
ZIP–Load	Type-I	2470	–	–	108.0	8693	49.6	–
	Type-II	–	1210	–	149.0	11994	–	15.5
	Type-III	–	–	2993 (0.82)	67.0	5393	49.6	9.3
	Simultaneous Placement	2470	1210	–	57.0	4588	49.6	5.0

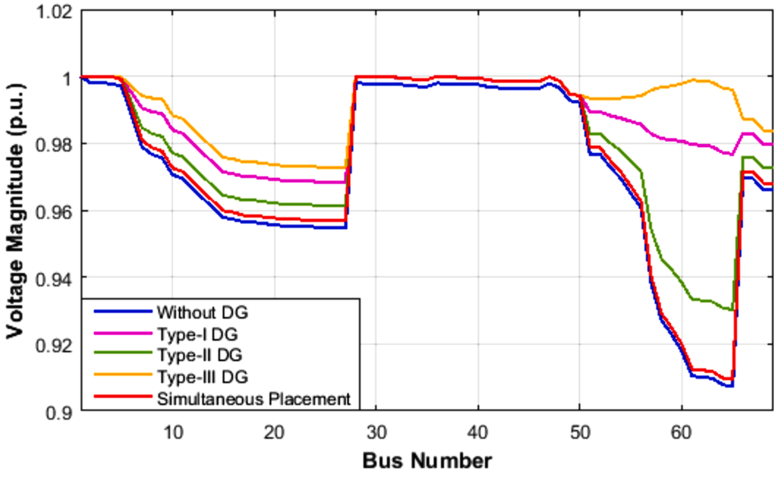

Fig. 7. Voltage profile of various DG installations for IEEE-69 test-bed.

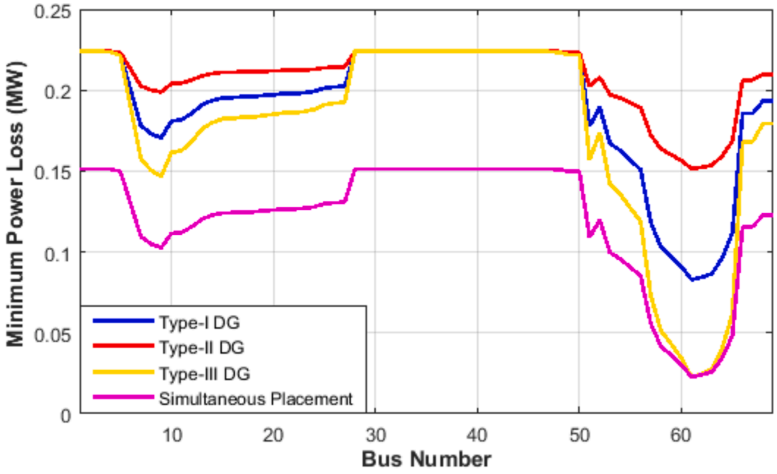

Fig. 8. Total power loss for IEEE-69 test-bed.

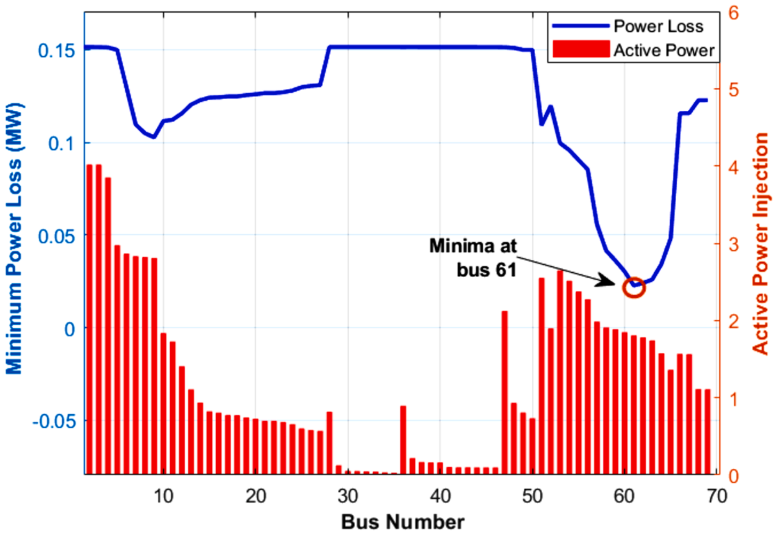

Fig. 9. Optimal location for IEEE-69 test-bed under ZIP load scenario.

Table 4
Optimal sizing and monetary benefit analysis for IEEE-69 test-bed.

Load Scenario	DG Type	Optimal DG Size kW	kVAR	kVA (NOPF)	Active Power Loss (kW)	Cost of Energy Losses ($)	P_G Cost ($/MWh)	Q_G Cost ($/MVARh)
CP–Load	Type-I	1810	–	–	83.3	6705	38.7	–
	Type-II	–	1290	–	152.1	12243	–	16.5
	Type-III	–	–	2222 (0.81)	23.1	1859	38.7	7.3
	Simultaneous Placement	1810	1290	–	23.1	1859	38.7	7.3
CI–Load	Type-I	1800	–	–	81.6	6568	38.5	–
	Type-II	–	1290	–	149.9	12066	–	16.5
	Type-III	–	–	2214 (0.81)	21.8	1755	38.5	7.3
	Simultaneous Placement	1800	1290	–	21.8	1755	38.5	7.3
CZ–Load	Type-I	1800	–	–	80.1	6448	38.5	–
	Type-II	–	1290	–	148.0	11913	–	16.5
	Type-III	–	–	2214 (0.81)	20.6	1658	38.5	7.3
	Simultaneous Placement	1800	1290	–	20.6	1658	38.5	7.3
ZIP–Load	Type-I	1810	–	–	82.8	6665	38.7	–
	Type-II	–	1290	–	151.4	12187	–	16.5
	Type-III	–	–	2222 (0.81)	22.7	1827	38.7	7.3
	Simultaneous Placement	1810	1290	–	22.7	1827	38.7	7.3

Table 5
Comparative analysis with existing techniques.

Methodology	Optimal DG Location	Optimal DG Size (kW)	Active Power Loss with DG (kW)
PSO [4]	6	3150	115.2
LSF [11]	18	743	146.8
Hybrid-Approach [13]	6	2490	111.17
BSOA [16]	8	1857.5	118.1
CPLS [31]	8	1800	118.1
NM [31]	6	2494.8	111.14
FPA [33]	6	2300	112.2
GA [34]	6	2380	132.64
Proposed	6	2491	111.1

scenario are presented in Table 4.

4.3. Comparative analysis

A comparative analysis with existing technique is presented in Table 5. For fair comparison, IEEE-33 bus system with constant power load model is utilized for this analysis.

5. Conclusion

This article proposes an analytical approach based optimized installation of DGs under different loading scenarios. Various DG configurations and loadings have been considered to simulate real-life scenarios. The effectiveness of proposed methodology is tested on IEEE-33 and 69 bus systems. The results indicated that approach presented in this article has acceptable performance for finding optimal sizing and location of different characteristic DGs. Benefits in monetary terms and loss reduction verifies the effectiveness of the proposed methodology. Observation indicates that DG configuration Type-III shows the most effectiveness in loss reduction and improving voltage profile compared to Type I and Type II DGs. Similarly, it is also observed that the maximum benefit in terms of monetary and loss minimization has been achieved in CZ load model compared to CP, CI, and ZIP load models. Proposed approach always converges to optimal solutions in different loading scenarios and exhibits improvement in loss reduction and voltage profile. Comparative analysis also reflects the superiority of the proposed methodology.

CRediT authorship contribution statement

Mohan Kashyap: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation,

Writing – original draft, Writing – review & editing, Visualization, Project administration. **Satish Kansal:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision. **Rishabh Verma:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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