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Hydrothermal unit commitment with AC constraints by a new solution method based on benders decomposition

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ABSTRACT

This paper presents a new approach based on Benders decomposition (BD) to solve hydrothermal unit commitment problem with AC power flow and security constraints. The proposed method decomposes the problem into a master problem and two sets of sub-problems. The master problem applies integer programming method to solve unit commitment (UC) while the sub-problems apply nonlinear programming solution method to determine economic dispatch for each time period. If one sub-problem of the first set becomes infeasible, the corresponding sub-problem of the second set is called. Moreover, strong Benders cuts are proposed that reduce the number of iterations and CPU time of the Benders decomposition method. All constraints of the hydrothermal unit commitment problem can be completely satisfied with zero penalty terms by the proposed solution method. The methodology is tested on the 9-bus and IEEE 118-bus test systems. The obtained results confirm the validity of the developed approach.

1. Introduction

Unit commitment (UC) is a key operational function for today power systems. In regulated power systems, it determines the start-up and shutdown times and generation of units such that the system operating cost over the scheduling horizon (e.g., a day) is minimized and unit constraints and system constraints, such as generation limits, ramping limits, minimum up/down time constraints, system load balance constraint and spinning and operating reserve requirements are satisfied. UC plays a major role in the daily operation planning of electric power systems and many solution methods to solve this problem have been presented in the literature [1–4].

Many UC research works have only considered thermal unit commitment (TUC). However, lots of practical power systems have considerable amounts of hydro generators in addition to thermal units and hydro energy constitutes an important component in the supply mix. For instance, total installed capacity of Portugal power system in 2009 reached 16,738 MW, of which 4578 MW (27%) corresponded to hydro plants [5]. As another instance, hydro plants constitute more than 20% of total installed capacity of Iran's power system. Thus, UC model should include both TUC and hydro unit commitment (HUC) together to correctly schedule the operation of units in these power systems. In other words, hydrothermal unit commitment (HTUC) problem should be solved for the power systems. For this purpose, hydro unit constraints (such as turbine outflow limits, reservoir limits and water dynamic balance constraint) should be considered in the HTUC model in addition to the thermal unit and system constraints. Thus, HTUC is usually a large-scale, mixed-integer and nonlinear optimization problem for practical power systems.

Considering importance and complexity of HTUC problem, several methods have been proposed to solve this problem in recent years. In [6], an augmented Lagrangian method is used to solve the hydrothermal scheduling problem considering generation-load balance, spinning reserve, emission bounds and line flow limits. A multistage Benders decomposition method is presented in [7] to solve short-term hydrothermal scheduling problem in which the scheduling problem is modeled with a continuous formulation and a cost minimization objective function. Also, Benders decomposition methods are presented in [8–11] to solve security constrained hydrothermal scheduling problem. The DC model losses for each line are represented in [8] by a piecewise linear function. Not only the DC transmission losses but also the AC power flow constraints were considered in [9–11]. AC network modeling within the HTUC problem was studied in these references.

Combination of Branch and Bound (B&B) and quadratic programming (QP) is presented in [12] to solve security constrained unit commitment (SCUC) problem. However, hydro units are not considered in [12]. An algorithm is presented in [13] to deal with HTUC problem by improving B&B search method. In order to achieve this goal, an initial feasible integer solution is provided to lead the B&B to the optimal solution. In [14], short term hydroelectric scheduling is formulated as a network flow optimization model and solved by interior point (IP) methods. The primal–dual and predictor–corrector versions of such IP methods are developed

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Nomenclature

T	scheduling horizon (h)
N, H	number of thermal and hydro units, respectively
I	number of buses in the system
L	number of branches in the system
S_k^n	set of thermal units connected to bus k
S_k^h	set of hydro units connected to bus k
$F^n(P_t^n)$	fuel cost of thermal unit n as a function of its generation
A^n, B^n, C^n	fuel cost function coefficients of thermal unit n
SU^n	startup cost of thermal unit n
SD^n	shut down cost of thermal unit n
$Pload_t^k, Qload_t^k$	active (MW) and reactive (MVAR) loads on bus k at time t, respectively
SR_t	system required spinning reserve at time t (MW)
S_t^n, S_t^h	spinning reserve contribution of thermal and hydro units at time t, respectively
$S_n^{\max}$	maximum response rate constrained spinning reserve contribution of thermal unit n
$P_{\min}^n, Q_{\min}^n$	minimum active (MW) and reactive (MVAR) power outputs of thermal unit n, respectively
$P_{\max}^n, Q_{\max}^n$	maximum active (MW) and reactive (MVAR) power outputs of thermal unit n, respectively
UR^n	ramp-up rate limit of thermal unit n (MW/h)
DR^n	ramp-down rate limit of thermal unit n (MW/h)
T_n^{ON}, T_n^{OFF}	minimum up and down time of thermal unit n (hr), respectively
$P_{n,t}^{r,\min}, P_{n,t}^{r,\max}$	ramp rate constrained minimum and maximum active power output of thermal unit n at time t (MW), respectively
$P_{\max}^h, Q_{\max}^h$	maximum active (MW) and reactive (MVAR) power outputs of hydro unit h, respectively
$P_{\min}^h, Q_{\min}^h$	minimum active (MW) and reactive (MVAR) power outputs of hydro unit h, respectively
G_{ik}, B_{ik}	conductance and susceptance between bus i and bus k, respectively
$V_{\min}^i, V_{\max}^i$	minimum and maximum voltage magnitude limit at bus i (pu), respectively
$BF_{\max}^l$	maximum flow limit for branch l (MVA)
$Vol_{\min}^h, Vol_{\max}^h$	minimum and maximum storage volume (m^3) of reservoir h, respectively
$qout_{\min}^h, qout_{\max}^h$	minimum and maximum water discharge rates (m^3/h) of reservoir h, respectively
Vol_{end}^h	Specified storage volume (m^3) of reservoir h at the end of scheduling horizon
UP_h	total number of upstream units which are immediately above the reservoir h
u_t^n	commitment state of thermal unit n at time t (1 = ON, 0 = OFF)
P_t^n, Q_t^n	active (MW) and reactive (MVAR) power generation of thermal unit n at time t, respectively
θ_t^{ik}	difference between angles of buses i and k at time t
ν_t^i	voltage magnitude of bus i at time t (pu)
BF_t^l	power flow through branch l at time t (MVA)
r_t^h	commitment state of hydro unit h at time t (1 = ON, 0 = OFF)
P_t^h, Q_t^h	active (MW) and reactive (MVAR) power generation of hydro unit h at time t, respectively
Vol_t^h	water storage volume (m^3) of reservoir h at time t
$qout_t^h$	water discharge rate (m^3/h) of reservoir h at time t
In_t^h, Sh_t^h	natural inflow rate and spillage discharge rate (m^3/h) of reservoir h at time t, respectively
$\tau_{re,h}$	water transport delay from reservoir re to h
λ	a prefix indicating dual variable
$Z1_t$–$Z3_t$	penalty terms for constrains at time t
w	weight factor of the penalty terms.

and the resulting matrix structure is explored. Also, a dynamic programming approach is presented in Ref. [15] to solve the short-term scheduling problem of a hydropower plant that sells energy in a pool-based electricity market. Moreover, several evolutionary computation and artificial neural network techniques such as genetic algorithm (GA) [16], simulated annealing and peak-shaving methods [17], particle swarm optimization (PSO) [18], cultural algorithm [19], modified hybrid differential evolution (MHDE) algorithm [20], combination of differential evolution (DE) and sequential quadratic programming (SQP) [21], differential evolution with adaptive Cauchy mutation [22], chaotic hybrid differential evolution algorithm [23,24] and Hopfield neural network [25] have also been applied to solve short-term hydrothermal scheduling problem. A review of different optimization methods applied for solving this problem can be found in [26].

None of the research works reviewed above takes into account AC power flow constraints in its HTUC model. However, without considering AC power flow constraints, HTUC solution may lead to infeasible results deviating from nodal active/reactive power balance constraints. In this paper, a HTUC model incorporating both AC power flow and AC security constraints is presented. However, considering these constraints increases nonlinearity, nonconvexity and complexity of the HTUC formulation. Thus, a new solution method in the framework of Benders decomposition (BD) is proposed for efficiently solving the suggested HTUC model, which is another new contribution of the paper. The proposed solution method includes new strong Benders cuts and two sets of sub-problems with a new data flow among them to enhance the convergence behavior of the method and its ability for finding optimum feasible solutions.

The remaining parts of the paper are organized as follows. In Section 2, the problem formulation is presented. The proposed solution method to solve HTUC problem is introduced in Section 3. The obtained numerical results from the proposed method for the 9-bus and IEEE 118-bus test systems are presented and discussed in Section 4. Section 5 concludes the paper.

2. HTUC formulation

The objective function of the HTUC problem can be formulated as follows (all symbols of the paper are defined in nomenclature):

$$Cost = Min\left\{\sum_{t=1}^{T}\sum_{n=1}^{N}\left[\left(F^n(P_t^n)u_t^n\right) + Y_t^n + X_t^n\right]\right\} \quad (1)$$

The objective function includes the fuel, start-up and shutdown costs of thermal units. $F^n(P_t^n)$ is conventionally taken in a quadratic form:

$$F^n(P_t^n) = A^n \cdot (P_t^n)^2 + B^n \cdot P_t^n + C^n \quad n \in N, \ t \in T \quad (2)$$

In (1), Y_t^n and X_t^n represent startup and shutdown costs of thermal unit n at time t, respectively, which are determined based on the following inequalities:

$$Y_t^n \geqslant SU^n \cdot (u_t^n - u_{t-1}^n), \quad Y_t^n \geqslant 0 \ \ n \in N, \ t \in T \quad (3)$$

$$X_t^n \geqslant SD^n \cdot (u_{t-1}^n - u_t^n), \quad X_t^n \geqslant 0 \ \ n \in N, \ t \in T \quad (4)$$

Startup cost should be considered when the unit state switches from OFF status to ON status, which is modeled in the first inequality constraint of (3). The next inequality constraint of (3) avoids from entering negative values for the startup cost. Since the optimization model minimizes the objective function of (1), Y_t^n reaches its lower bound. In other words, Y_t^n becomes equal to SU^n when the unit state changes from OFF status to ON status (i.e., when $u_t^n - u_{t-1}^n = 1 - 0 = 1$) and becomes zero otherwise. Similarly, (4) models shutdown cost.

The constraints of the optimization problem are as follows:

– Minimum ON/OFF duration of thermal unit n:

$$\sum_{m=0}^{T_n^{ON}-1} u_{t+m}^n \geqslant T_n^{ON} \cdot (u_t^n - u_{t-1}^n) \quad n \in N,\ t \in T \tag{5}$$

$$\sum_{m=0}^{T_n^{OFF}-1} (1 - u_{t+m}^n) \geqslant T_n^{OFF} \cdot (u_{t-1}^n - u_t^n) \quad n \in N,\ t \in T \tag{6}$$

– Minimum and maximum active and reactive power outputs of hydraulic units

$$r_t^h \cdot P_{\min}^h \leqslant P_t^h \leqslant r_t^h \cdot P_{\max}^h \quad h \in H,\ t \in T \tag{7}$$

$$r_t^h \cdot Q_{\min}^h \leqslant Q_t^h \leqslant r_t^h \cdot Q_{\max}^h \quad h \in H,\ t \in T \tag{8}$$

P_t^h is considered to be a linear function of water discharge rate, which is the assumption adopted in several other research works in the area, such as [9–11,17,27].

– Water dynamic balance [28]

$$Vol_t^h = Vol_{t-1}^h + In_t^h - qout_t^h - Sh_t^h + \sum_{re=1}^{UP_h} \left(qout_{t-\tau_{re,h}}^{re} + Sh_{t-\tau_{re,h}}^{re} \right) \quad h \in H,\ t \in T \tag{9}$$

– Minimum and maximum volume of each reservoir

$$Vol_{\min}^h \leqslant Vol_t^h \leqslant Vol_{\max}^h \quad h \in H,\ t \in T \tag{10}$$

– Minimum and maximum water discharge rate

$$r_t^h \cdot qout_{\min}^h \leqslant qout_t^h \leqslant r_t^h \cdot qout_{\max}^h \quad h \in H,\ t \in T \tag{11}$$

– Final volume constraint

$$Vol_T^h = Vol_{end}^h \quad h \in H \tag{12}$$

– The ramp rate constrained operating region and reactive capacity limits of thermal units for each period $t \in T$ can be stated as follows:

$$P_{n,t}^{r,\min} = \max\{P_{\min}^n, P_{t-1}^n - DR^n\} \tag{13}$$

$$P_{n,t}^{r,\max} = \min\{P_{\max}^n, P_{t-1}^n + UR^n\} \tag{14}$$

$$u_t^n \cdot P_{n,t}^{r,\min} \leqslant P_t^n \leqslant u_t^n \cdot P_{n,t}^{r,\max} \tag{15}$$

$$u_t^n \cdot Q_{\min}^n \leqslant Q_t^n \leqslant u_t^n \cdot Q_{\max}^n \tag{16}$$

– Spinning reserve requirement

$$\begin{aligned} &S_t^n = \min\left(\left(P_{n,t}^{r,\max} - P_t^n \right),\ \left(u_t^n \cdot S_n^{\max} \right) \right) \\ &S_t^h = r_t^h \cdot P_{\max}^h - P_t^h \\ &\sum_{n=1}^{N} S_t^n + \sum_{h=1}^{H} S_t^h \geqslant SR_t \quad n \in N,\ h \in H,\ t \in T \end{aligned} \tag{17}$$

– Power balance constraint (AC power flow)
Nodal balance (active and reactive) on every bus $k \in I$ for every period $t \in T$ can be represented as follows:

$$\sum_{n \in S_k^n} P_t^n + \sum_{h \in S_k^h} P_t^h - Pload_t^k = v_t^k \sum_{i=1}^{I} v_t^i \left(G_{ki} \cos\theta_t^{ki} + B_{ki} \sin\theta_t^{ki} \right) \tag{18}$$

$$\sum_{n \in S_k^n} Q_t^n + \sum_{h \in S_k^h} Q_t^h - Qload_t^k = v_t^k \sum_{i=1}^{I} v_t^i \left(G_{ki} \sin\theta_t^{ki} - B_{ki} \cos\theta_t^{ki} \right) \tag{19}$$

– Security constraints [29]
Bus voltage limits:

$$V_{\min}^i \leqslant v_t^i \leqslant V_{\max}^i \quad i \in I,\ t \in T \tag{20}$$

Capacity limits of branches:

$$\left| BF_t^l \right| \leqslant BF_{\max}^l \quad l \in L,\ t \in T \tag{21}$$

The security constraints can also be considered in the post-contingent state of credible contingencies in addition to base case conditions to include the effect of the contingencies [29].

It is noted that the valve loading effects of thermal units, prohibited discharge zone (PDZ) constraints of hydro units and prohibited operating zone (POZ) constraints of thermal units are neglected here for the sake of simplicity and better illustration of the proposed solution method. However, an explanation about valve loading effects, POZ and PDZ constraints can be found in previous works, such as [28,29].

3. The proposed solution method

The HTUC formulation is a nonlinear model due to e.g., the objective function and AC power flow constraints. Also, the commitment states of hydro and thermal units are binary variables. Thus, the HTUC formulation is a mixed integer nonlinear programming (MINLP) model. In this paper, a new solution method based on BD is proposed to solve the HTUC problem. The proposed method solves in a cycle of iterations a mixed integer linear programming (MILP) problem (master problem) and relaxed nonlinear programming (NLP) problems (sub-problems) with fixed integer variables. The master problem deals with the binary variables u_t^n, r_t^h and continuous variables $P_t^h, \eta_{F,t}, \eta_{S,t}$. The parameters of $\eta_{F,t}$ and $\eta_{S,t}$ are Benders cuts that represent the sub-problems into the master problem. The sub-problems consist of the parts of the optimization problem including continuous variables. After solving the sub-problems, a set of dual variables will be obtained and the Benders cuts will be added to the master problem.

The proposed solution method to solve the HTUC problem includes two enhancements with respect to the original BD approach. In this paper, we use two sets of sub-problems with a new data flow and job division among them. In some previous research works using BD approach, two sub-problems have also been proposed such as [30,31] on the security constrained unit commitment (SCUC). However, these approaches divide the constraints between the two sub-problems. For instance, in [30], AC power flow constraints are considered in one sub-problem and security constraints are included in the other one. As another instance, in [31], load balance equation and transmission security limits are considered in a sub-problem and voltage security limits are included in the other one. On the other hand, a new formulation of sub-problems is proposed here in which all constraints are considered in the first set sub-problems with one sub-problem for each time period. If one of the first set sub-problems becomes infeasible, the corresponding sub-problem of the second set, including appropriate penalty terms, is called to make it feasible. Additionally, strong Benders cuts are proposed, which enhance the convergence behavior of the solution method. Also, to further expedite the convergence, two strong cuts, corresponding to the first and second set sub-problems, will be added for each time period to the master problem. In the following, mathematical details of the proposed solution method are presented.

3.1. Master problem model

The cost function of (1) is decomposed to the objective functions of the master problem and sub-problems. The master problem in the proposed solution method has MILP model. The following objective function is considered for it:

$$\mu_M = Min\left\{\sum_{t=1}^{T}\left[\sum_{n=1}^{N}((C^n \cdot u_t^n) + Y_t^n + X_t^n) + \eta_t\right]\right\} \tag{22}$$

The last term of (22), i.e. η_t, is obtained from the Benders cuts. Since the proposed formulation has two sets of sub-problems, instead of a single set of cuts η_t, two sets of cuts $\eta_{F,t}$ and $\eta_{S,t}$ (corresponding to the first and second set sub-problems, respectively) are taken into account in the objective function of the master problem as follows:

$$\mu_M = Min\left\{\sum_{t=1}^{T}\left(\sum_{n=1}^{N}[(C^n \cdot u_t^n) + Y_t^n + X_t^n] + \eta_{F,t} + \eta_{S,t}\right)\right\} \tag{23}$$

$\eta_{F,t}$ and $\eta_{S,t}$ will be introduced in the sub-problems' formulations. The constraints of the master problem include the inequalities (3) and (4), minimum ON/OFF duration of thermal units (5) and (6), minimum and maximum active power output of hydraulic units (7), and the constraints of the hydroelectric power plants (9)–(12). The remaining part of the production cost of (1) is considered as the objective function of the first set sub-problems. In other words, with the proposed decomposition strategy, the commitment decision variables of u_t^n and r_t^h and dispatch decision variables of P_t^h are determined in the master problem, while the dispatch decision variables of P_t^n are considered in the sub-problems. Thus, with this decomposition strategy, the master problem saves its linearity and the first set sub-problems will have degrees of freedom, including P_t^n decision variables, to satisfy their own constraints, such as AC power flow constraints. The formulations of the first and second set sub-problems are introduced in the next subsections, respectively.

3.2. First set sub-problems

In the first sub-problem, an economic dispatch algorithm allocates power generation among committed thermal units. The objective function of the first sub-problem is as follows (the remaining part of the production cost of (1)):

$$\mu_F = Min\left\{\sum_{t=1}^{T}\sum_{n=1}^{N}\left[\left(B^n \cdot p_t^n + A^n \cdot (p_t^n)^2\right) \cdot u_t^n\right]\right\} \tag{24}$$

It is noted that u_t^n variables have been already determined in the master problem and so the sub-problem is a NLP optimization problem not mixed integer one.

The minimization of the NLP sub-problem is subject to reactive power limits of hydraulic units (8), ramp rate constrained operating region and reactive capacity limits of thermal units (13)–(16), spinning reserve requirement (17), nodal active and reactive power balance constraints (18) and (19), bus voltage limits (20) and capacity limits of branches (21).

Also, the decision variables determined in the master problem are input data to the first sub-problem:

$$u_t^n = \bar{u}_t^n \tag{25}$$

$$r_t^h = \bar{r}_t^h \tag{26}$$

$$P_t^h = \bar{P}_t^h \tag{27}$$

where $\bar{u}_t^n, \bar{r}_t^h$ and $\bar{P}_t^h$ are the obtained results from the master problem. Therefore, the first sub-problem can be decomposed in time to a set of hourly sub-problems (one sub-problem for each hour), which are solved considering the ramp rate constraints (13) and (14) as well as the other sub-problem's constraints mentioned above. Thus, the hourly decomposition approach provides feasible solutions (if existent). After the decomposition, the objective function (24) for each sub-problem t becomes as follows:

$$\mu_{F,t} = Min\left\{\sum_{n=1}^{N}\left[B^n \cdot P_t^n + A^n \cdot (P_t^n)^2\right]\right\} \tag{28}$$

The multiplication with u_t^n variable, as seen in (24), is not considered in (28) due to constraint (15). It is noted that the hourly decomposition approach might find slightly higher cost for the sub-problem compared with solving the sub-problem as a whole, but our experience shows that the difference between the two approaches (if exists) is usually very low. Thus, the generated Benders cuts using the hourly decomposition approach still provide a lower approximation of the true cost. On the other hand, solution times of the first and second sub-problems with hourly decomposition approach are much lower than the solution times of these sub-problems without the hourly decomposition. Moreover, without the hourly decomposition, Benders cuts are generated considering the feasibility status of the whole sub-problem, while the hourly decomposition allows us to generate more accurate Benders cuts separately considering the feasibility status of each hour.

If an hourly sub-problem of the first set becomes feasible, the following Benders cut will be added to the master problem in the next iteration:

$$\eta_{F,t} \geqslant \mu_{F,t} + \sum_{n\in N}\lambda u_{F,t}^n(u_t^n - \bar{u}_t^n) + \sum_{h\in H}\lambda r_{F,t}^h(r_t^h - \bar{r}_t^h) + \sum_{h\in H}\lambda P_{F,t}^h\left(P_t^h - \bar{P}_t^h\right) \tag{29}$$

where $\mu_{F,t}$ is as defined in (28) and $\lambda u_{F,t}^n, \lambda r_{F,t}^h$ and $\lambda P_{F,t}^h$ are dual variables or Lagrange multipliers corresponding to the constraints (25)–(27) of the sub-problems, respectively. In the previous research works in the area, such as [9–11,27], $\bar{u}_t^n, \bar{r}_t^h$ and $\bar{P}_t^h$ are obtained from the master problem solution in the previous iteration and $\mu_{F,t}$, $\lambda u_{F,t}^n, \lambda r_{F,t}^h$ and $\lambda P_{F,t}^h$ are obtained from the sub-problem solution in the previous iteration. Here, this cut is called as normal cut. However, in this paper, a new cut, called strong cut, is introduced as follows:

$$\eta_{F,t} \geqslant \mu_{F,t,b} + \sum_{n\in N}\lambda u_{F,t,b}^n\left(u_t^n - \bar{u}_{t,b}^n\right) + \sum_{h\in H}\lambda r_{F,t,b}^h\left(r_t^h - \bar{r}_{t,b}^h\right) + \sum_{h\in H}\lambda P_{F,t,b}^h\left(P_t^h - \bar{P}_{t,b}^h\right) \tag{30}$$

where

$$\mu_{F,t,b} + \sum_{n\in N}\lambda u_{F,t,b}^n\left(u_t^n - \bar{u}_{t,b}^n\right) + \sum_{h\in H}\lambda r_{F,t,b}^h\left(r_t^h - \bar{r}_{t,b}^h\right) + \sum_{h\in H}\lambda P_{F,t,b}^h\left(P_t^h - \bar{P}_{t,b}^h\right) = \underset{\substack{\text{All earlier iterations}\\ \text{in which the first set}\\ \text{sub-problem } t \text{ has}\\ \text{become feasible}}}{Max}\left[\begin{array}{l}\mu_{F,t} + \sum_{n\in N}\lambda u_{F,t}^n(u_t^n - \bar{u}_t^n)\\ + \sum_{h\in H}\lambda r_{F,t}^h(r_t^h - \bar{r}_t^h)\\ + \sum_{h\in H}\lambda P_{F,t}^h\left(P_t^h - \bar{P}_t^h\right)\end{array}\right] \tag{31}$$

In other words, the subscript b indicates iteration with the highest value of

$$\mu_{F,t} + \sum_{n\in N}\lambda u_{F,t}^n(u_t^n - \bar{u}_t^n) + \sum_{h\in H}\lambda r_{F,t}^h(r_t^h - \bar{r}_t^h) + \sum_{h\in H}\lambda P_{F,t}^h\left(P_t^h - \bar{P}_t^h\right)$$

among all earlier iterations in which the first set sub-problem t has become feasible. This iteration b instead of the previous iteration is used to generate strong Benders cut for the feasible sub-problem t of the first set. The proposed strong Benders cut in (30) and (31) is relatively similar to Pareto-optimal cut [32] considering that no other cut among the earlier iterations can dominate the strong cut, while there is no such guarantee for the normal cuts generated only based on the previous iteration. The proposed strong cuts can enhance the convergence of the Benders decomposition method. It is noted that for each sub-problem t, a separate iteration b can be selected. Moreover, even if the same iteration b is selected for a sub-problem $t1$ in two successive iterations among their earlier ones, different strong Benders cuts $\eta_{F,t1}$ will be added to the master problem in the two iterations, since u_t^n, r_t^h and P_t^h in (30) are continuously updated in the master problem. In other words, the proposed strong Benders cuts save their dynamic behavior along the iterations.

If an hourly sub-problem of the first set becomes infeasible, the strong Benders cut $\eta_{F,t}$, described in (30) and (31), for this hour is added to the master problem of the next iteration as well, however, the '*Max*' operator in (31) does not include the current iteration despite the previous case (feasible sub-problem). Besides, for the infeasible first set sub-problem, the corresponding hourly sub-problem of the second set is called to transform the infeasible NLP problem to a feasible one. The formulation of the second set sub-problems is introduced in the next subsection.

3.3. Second set sub-problems

If the first set sub-problem for hour t becomes infeasible, its corresponding sub-problem of the second set is defined as follows:

$$\mu_{S,t} = \min\left\{\sum_{n=1}^{N} Z1_t^n + \sum_{i=1}^{I} Z2_t^i + \sum_{l=1}^{L} Z3_t^l\right\} \tag{32}$$

$$Z1_t^n = \begin{cases} u_t^n \cdot P_{n,t}^{r,\min} - P_t^n & \text{if } P_t^n < u_t^n \cdot P_{n,t}^{r,\min} \\ P_t^n - u_t^n \cdot P_{n,t}^{r,\max} & \text{if } u_t^n \cdot P_{n,t}^{r,\max} < P_t^n \\ 0 & \text{Otherwise} \end{cases} \tag{33}$$

$$Z2_t^i = \begin{cases} V_{\min}^i - v_t^i & \text{if } v_t^i < V_{\min}^i \\ v_t^i - V_{\max}^i & \text{if } V_{\max}^i < v_t^i \\ 0 & \text{Otherwise} \end{cases} \tag{34}$$

$$Z3_t^l = \begin{cases} \left|BF_t^l\right| - BF_{\max}^l & \text{if } BF_{\max}^l < \left|BF_t^l\right| \\ 0 & \text{Otherwise} \end{cases} \tag{35}$$

The minimization of $\mu_{S,t}$ defined in (32), is subject to constraints (8), (13), (14), (16), (17), (18) and (19), which have no associated penalty term in the objective function (32). In other words, the constraints (8), (13), (14), (16), (17), (18) and (19) are directly imposed and deficit/excess of active/reactive power are appeared as the violation of the constraints (15), (20) and (21). These violations are penalized by the penalty terms $Z1_t^n$, $Z2_t^i$ and $Z3_t^l$ shown in (33)–(35). For instance, the reactive power limits of the generators (shown in (8) and (16)) are directly imposed and lack of sufficient reactive power sources (if existent) are appeared as the violation of the other constraints such as (20) penalized by $Z2_t^i$ terms [29].

The decision variables determined in the master problem are input data to the second set sub-problems like the first set sub-problems. In other words, the constraints (25)–(27) should also be considered for the second set sub-problems. After solving the second set sub-problem for hour t, the corresponding strong Benders cut, denoted by $\eta_{S,t}$, is added to the master problem of the next iteration like the strong Benders cut of the first set sub-problem, shown in (30) and (31). The strong Benders cut $\eta_{S,t}$ is defined as follows:

$$\eta_{S,t} \geqslant w\cdot\mu_{S,t,c} + \sum_{n\in N}\lambda u_{S,t,c}^n\left(u_t^n - \bar{u}_{t,c}^n\right) + \sum_{h\in H}\lambda r_{S,t,c}^h\left(r_t^h - \bar{r}_{t,c}^h\right) + \sum_{h\in H}\lambda P_{S,t,c}^h\left(P_t^h - \bar{P}_{t,c}^h\right) \tag{36}$$

where

$$w\cdot\mu_{S,t,c} + \sum_{n\in N}\lambda u_{S,t,c}^n\left(u_t^n - \bar{u}_{t,c}^n\right) + \sum_{h\in H}\lambda r_{S,t,c}^h\left(r_t^h - \bar{r}_{t,c}^h\right) + \sum_{h\in H}\lambda P_{S,t,c}^h\left(P_t^h - \bar{P}_{t,c}^h\right) = \underset{\substack{\text{All earlier iterations in which}\\ \text{the first set sub-problem } t \text{ has}\\ \text{become infeasible and the}\\ \text{corresponding sub-problem}\\ \text{of the second set is called}}}{Max}\left[\begin{array}{l} w\cdot\mu_{S,t} + \sum_{n\in N}\lambda u_{S,t}^n\left(u_t^n - \bar{u}_t^n\right) \\ + \sum_{h\in H}\lambda r_{S,t}^h\left(r_t^h - \bar{r}_t^h\right) \\ + \sum_{h\in H}\lambda P_{S,t}^h\left(P_t^h - \bar{P}_t^h\right) \end{array}\right] \tag{37}$$

Here, $\lambda u_{S,t}^n, \lambda r_{S,t}^h$ and $\lambda P_{S,t}^h$ are dual variables for the second set sub-problem t like $\lambda u_{F,t}^n, \lambda r_{F,t}^h$ and $\lambda P_{F,t}^h$ for the first set sub-problem t. The subscript c for the strong Benders cut $\eta_{S,t}$ in (36) is similar to the subscript b for the strong Benders cut $\eta_{F,t}$ in (30). The iteration c instead of previous iteration is used to generate Benders cut for the sub-problem t of the second set.

It is noted that, if an hourly sub-problem of the first set in an iteration of the proposed Benders decomposition method becomes feasible, the corresponding hourly sub-problem of the second set is not called. However, the strong Benders cut $\eta_{S,t}$ for this hour is added to the master problem of the next iteration as well, but in this case, the '*Max*' operator in (37) does not include the current iteration with feasible state for the hourly first set sub-problem. The addition of strong Benders cut $\eta_{S,t}$, besides $\eta_{F,t}$, for feasible first set hourly sub-problems avoids the recurrence of infeasibility for the first set sub-problems that have become feasible, which enhances the convergence of the proposed Benders decomposition method.

Considering the above explanation and the last paragraph of Section 3.2, it is seen that both the proposed strong Benders cuts $\eta_{F,t}$ and $\eta_{S,t}$ are added to the master problem of the next iteration for each time period regardless of the feasibility of its first set sub-problem, although the range of '*Max*' operator in (31) and (37) changes. This characteristic of the strong Benders cuts are also effective in the convergence of the proposed BD approach.

The proposed solution method is a BD approach with memory such that the information of all previous iterations are used to generate more effective strong Benders cuts. The proposed strong Benders cuts retain the characteristic of normal cuts without eliminating any part of the true cost. However, instead of previous iteration in normal cuts, an iteration is selected among all earlier ones that produces the strongest cut. Up to the authors' knowledge, the strong Benders cuts are specific to this work and have not been presented in the previous research works in the area.

3.4. Solution procedure

Application of the proposed BD approach for solution of HTUC problem with AC constraints can be summarized as the following step by step algorithm:

(1) Initialize parameters of the proposed BD approach including iteration number (*Iter*), upper bound (*UB*) and lower bound (*LB*) [32]: *Iter* = 1, *UB* = ∞, *LB* = $-\infty$.
(2) Solve the master problem with $\eta_{F,t} = \eta_{S,t} = 0$, $t = 1,\ldots,T$, which can be interpreted as the master problem without the Benders cuts. The objective function is μ_M defined in (23) and the constraints include (3)–(7) and (9)–(12). From the solution of this master problem u_t^n, r_t^h and P_t^h are obtained.

(3) Update LB of the BD: $LB = \mu_M - \sum_{t=1}^{T}\eta_{S,t}$. It is noted that μ_M includes both the strong cuts $\eta_{F,t}$ and $\eta_{S,t}$ as shown in (23). However, only $\eta_{F,t}$ contains a part of the cost function, while $\eta_{S,t}$ includes the penalty terms. So, $\eta_{S,t}$ terms are removed from LB, which is used for the calculation of duality gap. As shown in the next steps, the penalty terms' condition is separate from the duality gap condition.
(4) Solve the first set sub-problems. The objective function of the first set sub-problem t (t = 1, 2,..., T) is shown in (28) subject to constraints (8), (13)–(21), and (25)–(27). If the sub-problem t becomes feasible, $\mu_{F,t}, \lambda u^n_{F,t}, \lambda r^h_{F,t}$ and $\lambda P^h_{F,t}$ are obtained; otherwise, the corresponding sub-problem of the second set is called (owning the objective function of (32) and constraints (8), (13), (14), (16)–(19), and (25)–(27) and by solving this sub-problem $\mu_{S,t}, \lambda u^n_{S,t}, \lambda r^h_{S,t}$ and $\lambda P^h_{S,t}$ are obtained.
(5) If all hourly sub-problems of the first set become feasible (the penalty terms of each hour are zero), go to the next step; otherwise, go to step 7.
(6) Update UB: $UB = \mu_M + \sum_{t=1}^{T}\mu_{F,t} - \sum_{t=1}^{T}\eta_{F,t} - \sum_{t=1}^{T}\eta_{S,t}$. Indeed, UB includes the cost function of the HTUC without the Benders cuts. With the current values of LB (updated in step 3) and UB, the duality gap condition $\frac{|UB-LB|}{|LB|} \leqslant \varepsilon$ is checked, where ε indicates the duality gap tolerance. If this condition is satisfied the proposed BD algorithm is terminated; otherwise go to the next step.
(7) Increment the iteration number (*Iter* = *Iter* + 1). Add both the strong Benders cuts $\eta_{F,t}$ and $\eta_{S,t}$ for all time periods to the master problem of the next iteration. Solve this master problem and update the decision variables of u^n_t, r^h_t and P^h_t. Go back to step 3.

As seen from step 2 of the algorithm, both the $\eta_{F,t}$ and $\eta_{S,t}$ are set to zero at the beginning of the proposed BD method. So, if a first set sub-problem t has been infeasible up to current iteration, $\eta_{F,t}$ remains zero for it. However, by encountering the first feasible state for the first set sub-problem t, $\eta_{F,t}$ for this hour, as shown in (30) and (31), will be added to the master problem of the next iteration. We have a similar discussion for $\eta_{S,t}$.

4. Numerical results

In order to show the effectiveness of the proposed approach to solve HTUC problem with AC constraints, it is tested with the well-known 9-bus and IEEE 118-bus systems usually considered as test case in the previous research works in the area such as [29,30,33]. The 9-bus test system has three thermal units, one hydro unit, nine transmission lines, and three loads. IEEE 118-bus test system has 54 thermal units, seven hydro units, 186 transmission lines, and 91 loads. The required HTUC data of these test systems are obtained from Refs. [34,35].

Obtained results from the proposed BD approach for the 9-bus test system are presented in Table 1 and compared with the results of some other methods. The first two solution methods of Table 1, i.e. SBB and DICOPT, are two efficient solvers for MINLP optimization problems within the well-known Generalized Algebraic Modeling Systems (GAMSs) software package [36]. An explanation about these solvers can be found in [37]. SBB and DICOPT are not based on the Benders decomposition framework and so number of BD iterations and duality gap are not presented for these methods. The third solution approach of Table 1 is a well-organized generalized Benders decomposition (GBD) method proposed in [9,10]. The results of this method reported in Table 1 have been directly quoted from these references. The spinning reserve constraint in [9,10] is slightly different with respect to (17) of this paper. So, for the sake of a fair comparison, the spinning reserve constraint according to [9,10] is also used for the SBB, DICOPT and proposed BD method in the numerical experiment of Table 1. As seen from Table 1, the proposed BD approach obtains the lowest cost, defined in (1), with the lowest computation time revealing the capability of the proposed method to solve HTUC problem with AC constraints. In Table 1, the computation times of SBB, DICOT and proposed BD method are measured on the simple hardware set of a Pentium IV personal computer 2 GHz with 512 MB RAM and the computation time of the GBD has been measured on a similar hardware set [10]. Table 1 also shows that the proposed BD method has considerably lower number of iterations and duality GAP with respect to the GBD approach of Sifuentes and Vargas [9,10]. In the solution obtained by the proposed method, all penalty terms shown in (32) become zero and all constraints of the HTUC problem with AC constraints are satisfied. To implement the proposed BD approach, the master and sub-problems are solved by CPLEX and SNOPT solvers of GAMS software, respectively. In Table 2, detailed results of the proposed method for this test case are presented.

Evolution of UB, LB and duality gap along the iterations of the proposed BD method is represented in Table 3 and graphically shown in Fig. 1. Iteration 1, including some infeasible hours, is shown by grey color in Table 3. Another important advantage of the proposed BD method, observed in Table 3 and Fig. 1, is monotonic increase of LB and monotonic decrease of UB for both the infeasible and feasible iterations causing the fast convergence of the proposed method (the duality gap rapidly decreases). As a comparison, UB evolution in the GBD approach of Sifuentes and Vargas [9,10] represent large oscillations for this test case, resulting in more iterations to reach convergence even with a considerably higher value of duality gap shown in Table 1.

Obtained results from the proposed method for IEEE 118-bus test system are shown in Table 4. The first two benchmark methods of Table 1 (i.e., the MINLP solvers of DICOPT and SBB) cannot solve the HTUC problem with AC constraints on the large test case of the IEEE 118-bus system due to its large number of decision variables and constraints. Moreover, we could not find any research work solving HTUC problem with AC constraints on this test system so that we can compare our method with its approach. So, we compare our method with its simpler versions to highlight the effectiveness of the proposed enhancements presented in the previous section. The first approach reported in Table 4 is the proposed method without the first enhancement (two sets of sub-problems). In this approach, there is only one set of sub-problems and their objective functions include the linear and quadratic parts of thermal units' cost curve plus required penalty terms to make feasible the solution. It is noted that the strong Benders cuts (second enhancement) are used for this method of Table 4, since the purpose of this numerical experiment is evaluating the effectiveness of the first enhancement. As seen, the first approach of Table 4 leads to a higher cost with higher computation time and number of iterations indicating its slow convergence. By adding the first enhancement to this approach leading to the proposed method (reported in the second row of Table 4), lower value of the cost, computation time and number of iterations is obtained. For all methods of Table 4, the duality gap tolerance ε is set to 0.1%. Moreover, in

Table 1
Obtained results for the HTUC problem with AC constraints in the 9-bus test system.

Solution method	Cost	Time (s)	Number of iterations	Duality gap (%)
SBB	73,721	240	–	–
DICOPT	73,663	143	–	–
GBD [9,10]	73,435	57	15	0.3626
Proposed BD	73,418	31	8	0.06

Table 2
Generation of units and storage volume of the hydro unit for the 9-bus test system (proposed BD approach).

Hour	Generation of thermal unit 1 (MW)	Generation of thermal unit 2 (MW)	Generation of thermal unit 3 (MW)	Generation of hydro unit (MW)	Hydro unit storage volume at the end of each hour (1000 m^3)
1	70.3		78.8	104.8	242
2	74.4		82.3	79.1	258
3	74.5		82.4	79.1	274
4	70.6		78.9	75.9	293
5	71		79.3	76.4	311
6	73.9		81.9	90.4	317
7	72.8		81	91.1	321
8	68.6		77.3	104.2	314
9	68.5		77.3	104.2	306
10	74.9		82.8	103.5	299
11	74		82.1	103.8	292
12	73.1		81.3	116.7	273
13	76.4		84.2	102.7	267
14	73.2		81.3	102.6	261
15	71.9		81.2	104.4	253
16	73.2		91.3	103.1	247
17	73		81.2	102.4	241
18	73.5		81.6	99.6	238
19	70.8	50.5	79	104.6	230
20	74.1	53.1	82	108.7	218
21	74.3	53.2	82.2	109.4	206
22	72.2	51.5	80.3	103	200
23	70.3	49.9	78.5	75.2	220
24	72.5	51.3	80.3	52	260.7

Table 3
Evolution of the proposed BD method for the 9-bus test system.

Iteration	1	2	3	4	5	6	7	8
UB	166,527	90,953	77,034	75,117	73,771	73,553	73,482	73,418
LB	1657	27,789	37,858	65,691	71,436	72,939	73,304	73,374
Duality gap (%)	9949	227	103	14	3.3	0.8	0.2	0.06

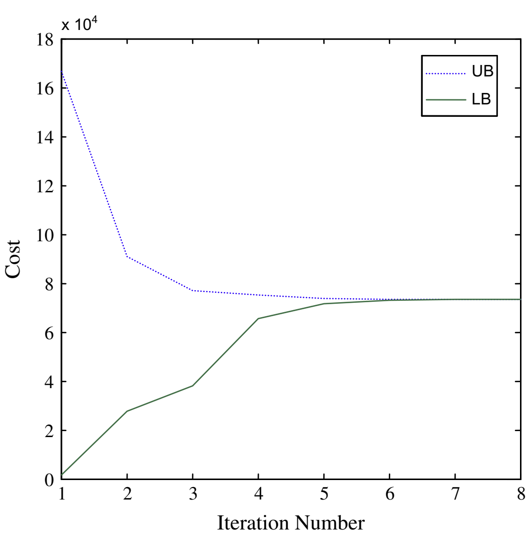

Fig. 1. Evolution of UB and LB of the proposed BD method for the 9-bus test system.

Table 4
Obtained results from the proposed BD method and its simpler version without the first enhancement for the IEEE 118-bus test system.

Solution method	Cost	Time (s)	Number of iterations
Proposed – first enhancement	867,130	915	25
Proposed	832,461	347	5

the final solution found by each of these approaches, the penalty terms become zero and all constraints of the HTUC problem with AC constraints are satisfied.

In addition to the alternative of Table 4, we also tested two additional decomposition strategies. In the first one, the objective function of the master problem is (22) plus the linear part of the fuel cost function of thermal units, i.e. $B^n \cdot P_t^n$ in (2). This decomposition strategy has been proposed in [11]. In the second one, both the linear and quadratic terms of the fuel cost function of thermal units, i.e. $B^n \cdot P_t^n$ and $A^n \cdot (P_t^n)^2$ in (2), are added to (22) such that the objective function of the master problem includes the whole cost function of thermal units. This decomposition strategy has been presented in some SCUC research works [30,31]. In both the strategies, the constraints of the active power limits of thermal units are also considered in the master problem. However, none of these two strategies results in a feasible solution with zero penalty terms for the HTUC problem with AC constraints on the IEEE 118-bus test case. We also included OPF decision variables (settings of phase-shifting transformers, tap-changing transformers and capacitor banks) in the optimization problem like [30]. However, again the two strategies could not reach zero penalty terms for the HTUC problem with AC constraints on the IEEE 118-bus test case. In these two strategies, the decision variables of P_t^n are determined in the master problem and so there is no decision variable (degree of freedom) to satisfy AC power flow constraints in the sub-problem (P_t^h variables have already been included in the master problem). Hence, satisfying AC power flow constraints becomes so hard (if not impossible).

We also performed numerical experiments to evaluate the effect of the second enhancement. However, without the strong cuts (i.e. by using the normal cuts), we could not find a feasible solution

Table 5
Evolution of the proposed BD method for the IEEE 118-bus test system.

Iteration	1	2	3	4	5
UB	1,432,196	1,103,264	994,521	845,213	832,461
LB	284,529	348,701	762,116	813,265	832,251
Duality gap (%)	403	216	30.5	3.9	0.025

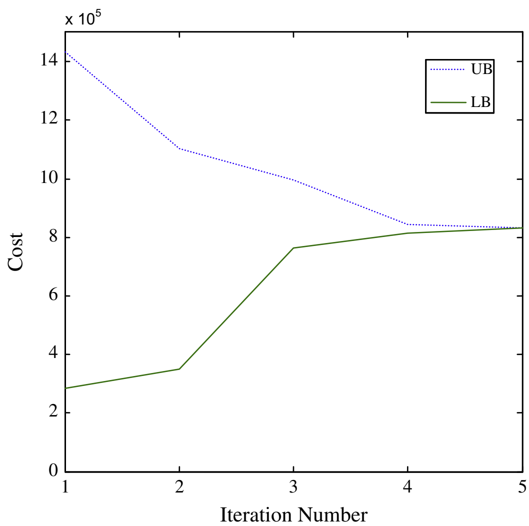

Fig. 2. Evolution of UB and LB of the proposed BD method for the IEEE 118-bus test system.

with zero penalty terms for the IEEE 118-bus test case and so the results of these experiments are not reported here. This indicates the effectiveness of the proposed strong cuts to solve the HTUC problem with AC constraints, which can provide a feasible optimum solution with zero penalty terms even for the large test case of the IEEE 118-bus test system.

Evolution of UB, LB and duality gap along the iterations of the proposed BD method, shown in the second row of Table 4, for the IEEE 118-bus test case is represented in Table 5 and graphically shown in Fig. 2. Iterations 1 and 2, including some infeasible hours, are shown by grey color in Table 5. As seen from Table 5 and Fig. 2, even for the large test case of the IEEE 118-bus system, UB/LB of the proposed BD method has monotonic decrease/increase, respectively, for both the infeasible and feasible iterations causing the fast convergence of the proposed method.

To numerically illustrate the difference between using the hourly decomposition strategy and solving the sub-problem as a whole, four additional experiments are performed on the 9-bus test system. This test system is selected, since the solvers of GAMS software package cannot solve the sub-problems on the IEEE 118-bus test system without the hourly decomposition. Obtained results from these four numerical experiments comparing the two approaches, i.e. without/with the hourly decomposition, are reported in Table 6. The first experiment uses the original data of the 9-bus test system that does not include the ramp rate limits, indicated by 'none' in the last column of the table. The next three experiments include ramp rate constraints and the applied ramp rate limits for the three thermal units of the 9-bus test system in each experiment are shown in the last column. Tighter ramp rate limits are considered for these three experiments from top to bottom. In the first three experiments of Table 6 that both the approaches can find feasible solutions, negligible differences are seen between the values of the cost objective function obtained by the two approaches. However, the computation times without the hourly decomposition strategy are much higher than the computation times with the hourly decomposition strategy, while the two approaches have approximately the same duality gap values. In the fourth experiment with the tightest ramp rate limits among the four experiments, without the hourly decomposition, no feasible solution is obtained at all, while with the hourly decomposition, feasible solution with reasonable computation time and duality gap is obtained. The numerical experiments of Table 6 further reveal the effectiveness of the hourly decomposition strategy applied for the sub-problems within the proposed BD method.

5. Conclusion

In this paper, the problem of HTUC with AC constraints is modeled. Most of previous research works in the area only focus on the HTUC problem, while their obtained schedules are usually infeasible because the nodal active/reactive power balance constraints and network security limits are not taken into account in the traditional HTUC models. Thus, the HTUC with AC constraints, which can actually provide a financially viable and physically feasible schedule, is considered to be one of best available options in power system operation. A new BD based solution method including important enhancements is proposed to solve this complex problem. The proposed method has two sets of sub-problems devoted to minimizing the remaining parts of the objective function (remained from the master problem) and penalty terms, respectively. Although some previous research works using BD approaches propose two sub-problems, their formulations and partitioning of tasks between the sub-problems are different with respect to our proposed approach. Additionally, strong Benders cuts are proposed to enhance the convergence behavior of the proposed BD method. Obtained results from extensive testing of the proposed method for solving the HTUC problem with AC constraints on the 9-bus and IEEE 118-bus test systems confirm the validity of the developed approach. The research work is under way in order to incorporate contingency constraints and stability considerations (such as voltage and transient stability constraints) into the proposed model for the HTUC problem with AC constraints. Also, extension of the for-

Table 6
Obtained results from the proposed BD method without/with the hourly decomposition for 9-bus test system with different ramp rate constraints.

Experiment	Solution method	Cost	Time (s)	Number of iterations	Duality gap (%)	Ramp rate limit (up and down)
1	Without[a]	73,412	80	9	0.07	None
1	With[b]	73,418	31	8	0.06	None
2	Without	73,414	97	9	0.07	150, 135, 125
2	With	73,424	33	8	0.05	150, 135, 125
3	Without	80,594	101	9	0.06	90, 80, 80
3	With	80,606	34	8	0.07	90, 80, 80
4	Without	Infeasible	–	–	–	80, 70, 70
4	With	84,103	37	9	0.07	80, 70, 70

[a] Without the hourly decomposition strategy.
[b] With the hourly decomposition strategy.

mulation to stochastic frameworks to model the uncertainty sources of the problem (such as inflow of hydro units, load forecast error and forced outage rate of units) will be considered in the future research.

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