We can solve for E'^r_{xfd} by first substituting (5.9-31) into (5.9-18), however, I^r_{ds} is required before E'^r_{xfd} can be evaluated. Thus from (5.9-12) we obtain

$$\begin{aligned} I^r_{ds} &= -\sqrt{2} I_s \sin[\theta_{ei}(0) - \theta_{ev}(0) - \delta] \\ &= -\sqrt{2}|\tilde{I}_{as}| \sin[-31.8^\circ - 0 - 18^\circ] \\ &= -\sqrt{2}(9.37 \times 10^3) \sin(-49.8^\circ) \\ &= 10.12\,\text{kA} \end{aligned} \tag{5A-4}$$

From (5.9-18) and (5.9-31) we have

$$\begin{aligned} E'^r_{xfd} &= \frac{\omega_e}{\omega_b}\left[\sqrt{2}|\tilde{E}_a| + \frac{\omega_e}{\omega_b}(X_d - X_q) I^r_{ds}\right] \\ &= \sqrt{2}(15.2 \times 10^3) + (1.0467 - 0.5911)10.12 \times 10^3 \\ &= 26.1\,\text{kV} \end{aligned} \tag{5A-5}$$

Because r_s is small, T_e may be calculated by substitution into (5.9-32):

$$\begin{aligned} T_e &= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left[\frac{E'^r_{xfd}\sqrt{2}|\tilde{V}_{as}|}{(\omega_e/\omega_b)X_d}\sin\delta \right. \\ &\quad \left. + \left(\frac{1}{2}\right)\left(\frac{\omega_e}{\omega_b}\right)^{-2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)(\sqrt{2}|\tilde{V}_{as}|)^2 \sin 2\delta\right] \\ &= \left(\frac{3}{2}\right)\left(\frac{64}{2}\right)\left(\frac{1}{377}\right)\left\{\frac{(26.1 \times 10^3)(\sqrt{2})[(20 \times 10^3)/(\sqrt{3})]}{1.0467}\sin 18^\circ \right. \\ &\quad \left. + \left(\frac{1}{2}\right)\left(\frac{1}{0.5911} - \frac{1}{1.0467}\right)\left[\sqrt{2}\left(\frac{20 \times 10^3}{\sqrt{3}}\right)^2\right]\sin 36^\circ\right\} \\ &= 23.4 \times 10^6\,\text{N}\cdot\text{m} \end{aligned} \tag{5A-6}$$

5.10 DYNAMIC PERFORMANCE DURING A SUDDEN CHANGE IN INPUT TORQUE

It is instructive to observe the dynamic performance of a synchronous machine during a step change in input torque. For this purpose, the differential equations that describe the synchronous machine were programmed on a computer and a study was performed [4]. Two large machines are considered: a low-speed hydro turbine generator and a high-speed steam turbine generator. Information regarding each machine is given in Tables 5.10-1 and 5-10.2. In the case of hydro turbine generator, parameters are given for only one damper winding in the q axis. The reason for

Table 5.10-1 Hydro Turbine Generator

Rating: 325 MVA
Line-to-line voltage: 20 kV
Power factor: 0.85
Poles: 64
Speed: 112.5 r/min
Combined inertia of generator and turbine:
$J = 35.1 \times 10^6 \text{J} \cdot \text{s}^2$, or $WR^2 = 833.1 \times 10^6 \text{lbm} \cdot \text{ft}^2$ $H = 7.5$ s
Parameters in ohms and per unit:
$r_s = 0.00234\,\Omega$, 0.0019 pu
$X_{ls} = 0.1478\,\Omega$, 0.120 pu

$X_q = 0.5911\,\Omega, 0.480$ pu	$X_d = 1.0467\,\Omega$, 0.850 pu
	$r'_{fd} = 0.00050\,\Omega, 0.00041$ pu
	$X'_{lfd} = 0.2523\,\Omega$, 0.2049 pu
$r'_{kq2} = 0.01675\,\Omega$, 0.0136 pu	$r'_{kd} = 0.01736\,\Omega$, 0.0141 pu
$X'_{lkq2} = 0.1267\,\Omega$, 0.1029 pu	$X'_{lkd} = 0.1970\,\Omega$, 0.160 pu

denoting this winding as the $kq2$ winding rather than the $kq1$ winding will become clear in Chapter 7.

The computer traces shown in Figs. 5.10-1 and 5.10-2 illustrate the dynamic behavior of the hydro turbine generator following a step change in input torque from zero to 27.6×10^6 N·m (rated for unity power factor). The dynamic behavior

Table 5.10-2 Steam Turbine Generator

Rating: 835 MVA
Line-to-line voltage: 26 kV
Power factor: 0.85
Poles: 2
Speed: 3600 r/min
Combined inertia of generator and turbine:
$J = 0.0658 \times 10^6 \text{ J} \cdot \text{s}^2$, or $WR^2 = 1.56 \times 10^6 \text{ lbm} \cdot \text{ft}^2$ $H = 5.6$ s
Parameters in ohms and per unit:

$r_s = 0.00243\,\Omega, 0.003$ pu
$X_{ls} = 0.1538\,\Omega, 0.19$ pu

$X_q = 1.457\,\Omega, 1.8$ pu	$X_d = 1.457\,\Omega, 1.8$ pu
$r'_{kq1} = 0.00144\,\Omega, 0.00178$ pu	$r'_{fd} = 0.00075\,\Omega, 0.000929$ pu
$X'_{lkq1} = 0.6578\,\Omega$, pu 0.8125 pu	$X'_{lfd} = 0.1145\,\Omega, 0.1414$ pu
$r'_{kq2} = 0.00681\,\Omega, 0.00841$ pu	$r'_{kd} = 0.01080\,\Omega, 0.01334$ pu
$X'_{lkq2} = 0.07602\,\Omega, 0.0939$ pu	$X'_{lkd} = 0.06577\,\Omega, 0.08125$ pu

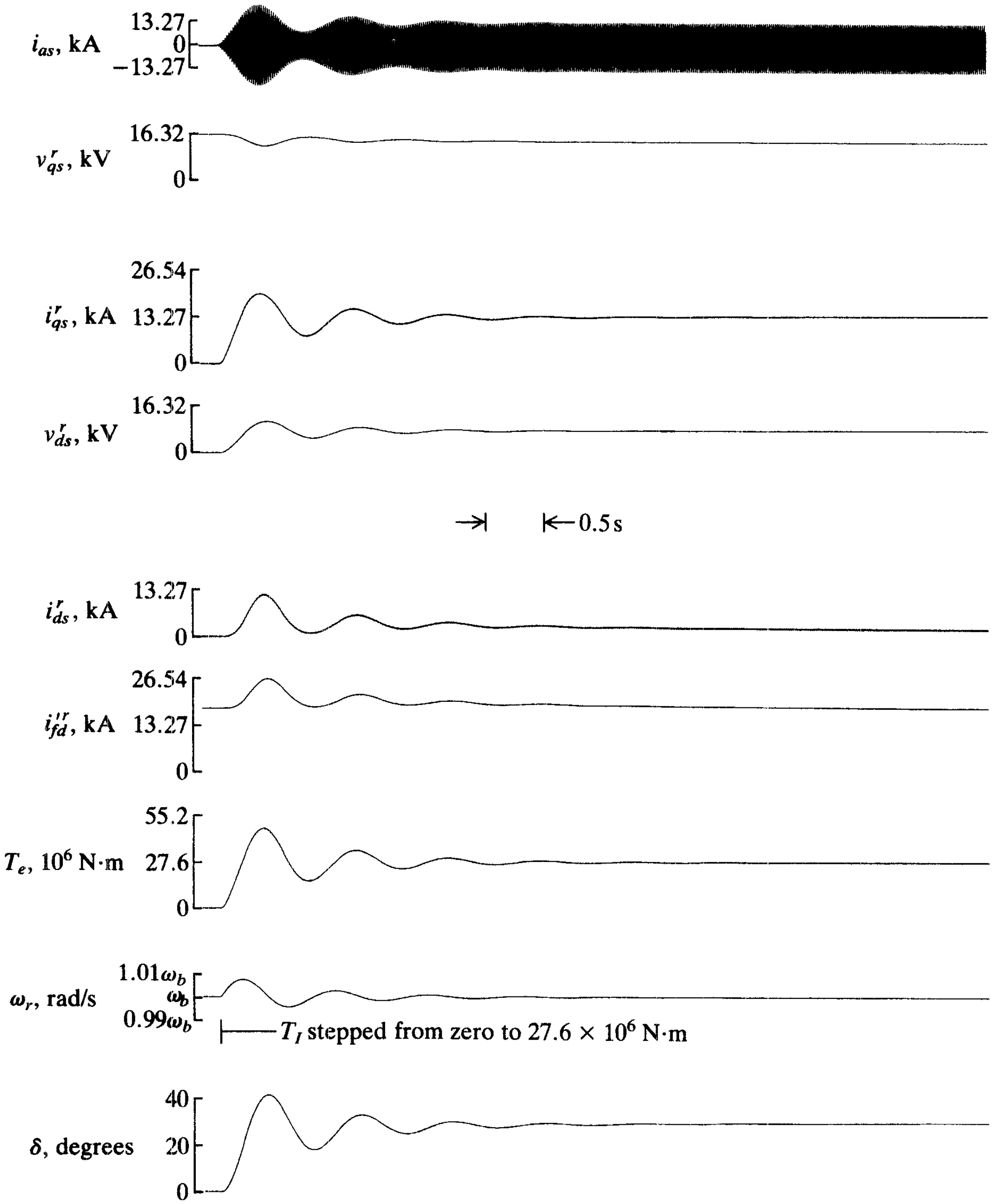

Figure 5.10-1 Dynamic performance of a hydro turbine generator during a step increase in input torque from zero to rated.

of the steam turbine generator is depicted in Figs. 5.10-3 and 5.10-4. In this case the step change in input torque is from zero to 1.11×10^6 N·m (50% rated). In Figs. 5.10-1 and 5.10-3 the following variables are plotted: i_{as}, v_{qs}^r, i_{qs}^r, v_{ds}^r, i_{ds}^r, $i_{fd}^{\prime r}$, T_e, ω_r, and δ, where ω_r is in electrical radians per second and δ is in electrical degrees. Figures 5.10-2 and 5.10-4 illustrate the dynamic torque versus rotor angle characteristics. In all figures, the scales of the voltages and currents are given in multiples of peak rated values.

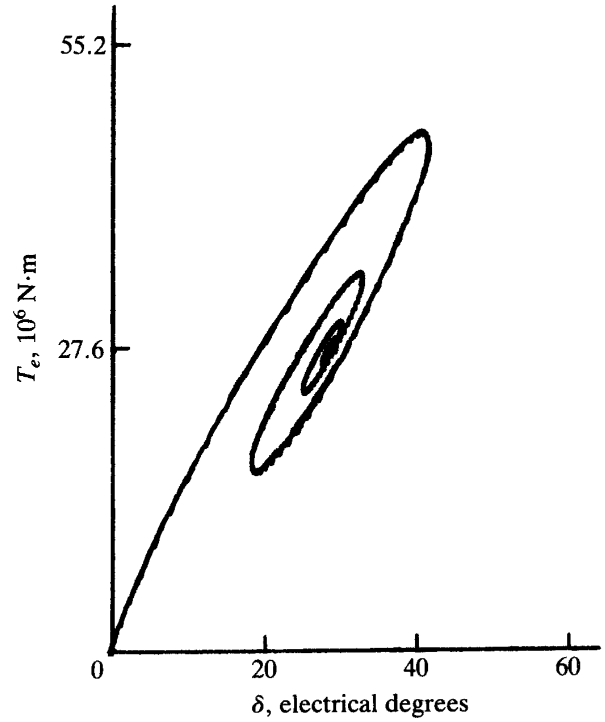

Figure 5.10-2 Torque versus rotor angle characteristics for the study shown in Fig. 5.10-1.

In each study, it is assumed that the machine is connected to a bus whose voltage and frequency remain constant, at the rated values, regardless of the stator current. This is commonly referred to as an infinite bus, because its characteristics do not change regardless of the power supplied or consumed by any device connected to it. Although an infinite bus cannot be realized in practice, its characteristics are approached if the power delivery capability of the system, at the point where the machine is connected, is much larger than the rating of the machine.

Initially each machine is operating with zero input torque with the excitation held fixed at the value that gives rated open-circuit terminal voltage at synchronous speed. It is instructive to observe the plots of T_e, ω_r, and δ following the step change input torque. In particular, consider the response of the hydro turbine generator (Fig. 5.10-1) where the machine is subjected to a step increase in input torque from zero to 27.6×10^6 N · m. The rotor speed begins to increase immediately following the step increase in input torque as predicted by (5.8-3), whereupon the rotor angle increases in accordance with (5.7-1). The rotor speeds up until the accelerating torque on the rotor is zero. As noted in Fig. 5.10-1, the speed increases to approximately 380 electrical radians per second, at which time T_e is equal to T_I because the change of ω_r is zero and hence the inertial torque (T_{IT}) is zero. Even though the accelerating torque is zero at this time, the rotor is running above synchronous speed; hence δ, and thus T_e, will continue to increase. The increase in T_e, which is an increase in the power output of the machine, causes the rotor to decelerate toward synchronous speed. However, when synchronous speed is reached, the magnitude of δ has become larger than necessary to satisfy the input torque. Note that at the first synchronous speed crossing of ω_r after the change in input torque, δ is

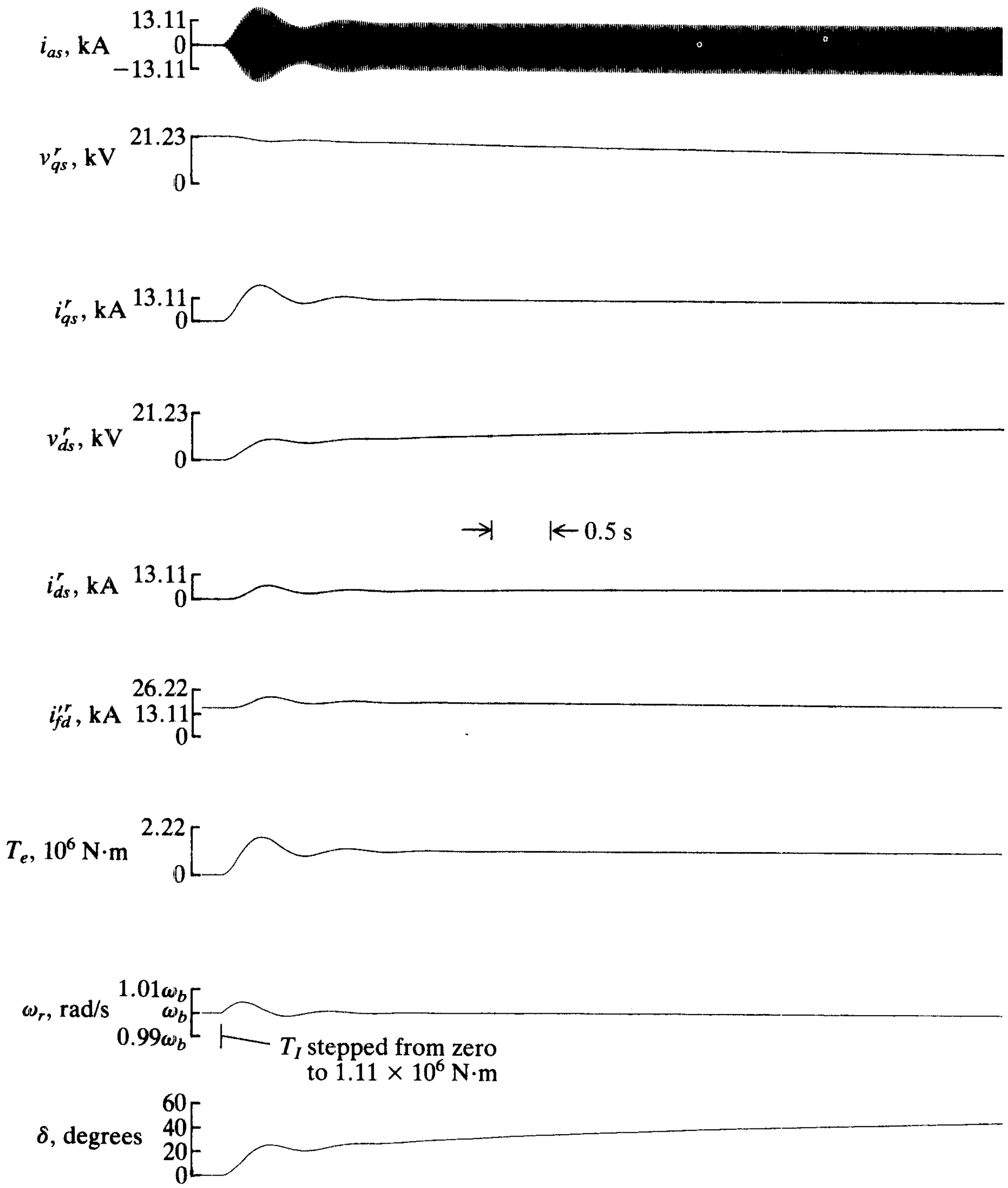

Figure 5.10-3 Dynamic performance of a steam turbine generator during a step increase in input torque from zero to 50% rated.

approximately 42 electrical degrees and T_e is approximately 47×10^6 N·m. Hence, the rotor continues to decelerate below synchronous speed and consequently δ begins to decrease, which in turn decreases T_e. Damped oscillations of the machine variables continue, and a new steady-state operating point is finally attained.

In the case of the hydro turbine generator (Fig. 5.10-1) the oscillations in machine variables subside in a matter of 2 or 3 s and the machine establishes the new steady-state operating point within 8 or 10 s. In the case of the steam turbine generator (Fig. 5.10-3) the oscillations subside rapidly, but the new steady-state operating

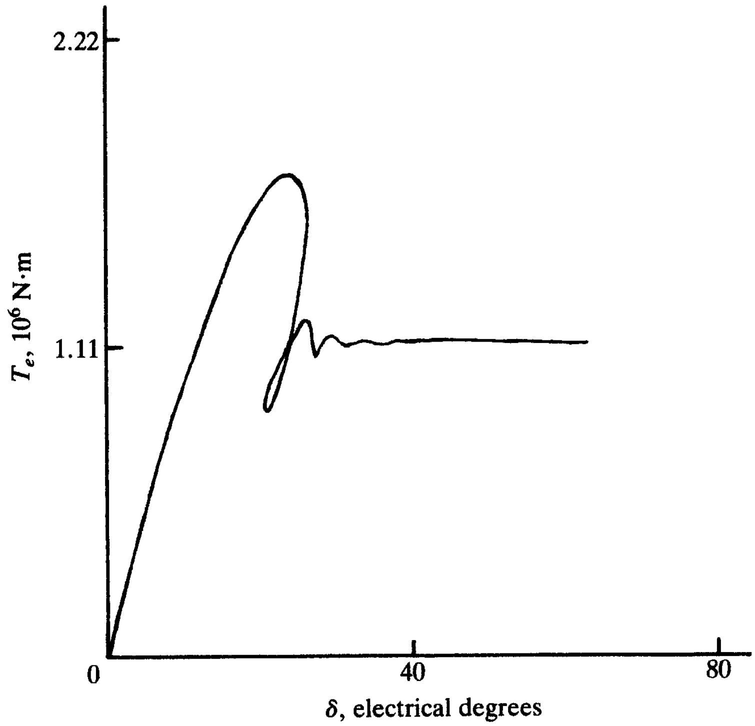

Figure 5.10-4 Torque versus rotor angle characteristics for the study shown in Fig. 5.10-3.

point is slowly approached. The damping is of course a function of the damper windings and can be determined from an eigenvalue analysis that will be discussed later. The point of interest here is the time required for the machine variables to reestablish steady-state operation after the torque disturbance. This rather slow approach to the new steady-state operating point in the case of the steam turbine generator is also apparent from the plot of T_e versus δ (Fig. 5.10-4).

Let us consider, for a moment, the expression for steady-state torque, (5.9-32). For the hydro turbine generator with $E'^{r}_{xfd} = \sqrt{\frac{2}{3}}\,20\,\text{kV}$ we have

$$T_e = (32.5\sin\delta + 12.5\sin 2\delta) \times 10^6\,\text{N}\cdot\text{m} \tag{5.10-1}$$

and for the steam turbine generator with $E'^{r}_{xfd} = \sqrt{\frac{2}{3}}\,26\,\text{kV}$ we obtain

$$T_e = 1.23 \times 10^6 \sin\delta\,\text{N}\cdot\text{m} \tag{5.10-2}$$

If (5.10-1) and (5.10-2) are plotted on Figs. 5.10-2 and 5.10-4, respectively, the steady-state T_e versus δ curves will pass through the final value of the dynamic T_e versus δ plots. However, the dynamic torque–angle characteristics immediately following the input torque disturbance yields a much larger T_e for a given value of δ than does the steady-state characteristic. In other words, the dynamic or transient torque–angle characteristic is considerably different from the steady-state characteristic and the steady-state T_e versus δ curve applies only after all transients have subsided. Although the computation of the transient torque during speed variations requires the solution of nonlinear differential equations, it can be approximated quite simply. This is the subject of a following section.

The studies shown in this section are for generator action. Motor action, wherein a load torque is applied to the machine, would essentially yield the mirror image of the T_e versus δ plots differing only by the ohmic losses.

5.11 DYNAMIC PERFORMANCE DURING A 3-PHASE FAULT AT THE MACHINE TERMINALS

The stability of synchronous machines throughout a power system following a fault is of major concern. A 3-phase fault or short-circuit rarely occurs, and a 3-phase fault at the machine terminals is even more uncommon; nevertheless, it is instructive to observe the dynamic performance of a synchronous machine during this type of a fault.

The computer traces shown in Figs. 5.11-1 and 5.11-2 illustrate the dynamic behavior of the hydro turbine generator during and following a 3-phase fault at the terminals. The dynamic behavior of the steam turbine generator as a result of a 3-phase terminal fault is shown in Figs. 5.11-3 and 5.11-4. The parameters of the machines are those given in the previous section. In Figs. 5.11-1 and 5.11-3 the following variables are plotted: i_{as}, v_{qs}^r, i_{qs}^r, v_{ds}^r, i_{ds}^r, $i_{fd}^{\prime r}$, T_e, ω_r, and δ. Figures 5.11-2 and 5.11-4 illustrate the dynamic torque–angle characteristics during and following the 3-phase fault.

In each case the machine is initially connected to an infinite bus delivering rated MVA at rated power factor. In the case of the hydro turbine generator the input torque is held constant at $(0.85)27.6\times10^6$ N·m with $E_{xfd}^{\prime r}$ fixed at $(1.6)\sqrt{\frac{2}{3}}20$ kV; for the steam turbine generator $T_I = (0.85)2.22\times10^6$ N·m and $E_{xfd}^{\prime r} = (2.48)\sqrt{\frac{2}{3}}26$ kV. (Rated operating conditions for the hydro turbine generator are calculated in Example 5A.) With the machines operating in this steady-state condition, a 3-phase terminal fault is simulated by setting v_{as}, v_{bs}, and v_{cs} to zero, in the simulation, at the instant v_{as} passes through zero going positive. The transient offset in the phase currents is reflected into the rotor reference-frame variables and the instantaneous torque as a decaying 60-Hz pulsation. Because the terminal voltage is zero during the 3-phase fault, the machine is unable to transmit power to the system. Hence, all of the input torque, with the exception of the ohmic losses, accelerates the rotor.

In the case of the hydro turbine generator the fault is removed in 0.466 s; 0.362 s in the case of the steam turbine generator. If the fault had been allowed to remain on the system slightly longer, the machines would have become unstable; that is, they would either not have returned to synchronous speed after removal of the fault or slipped poles before returning to synchronous speed. Asynchronous operation (pole slipping) is discussed in Chapter 10 of reference 5.

When the fault is cleared, the system voltages are reapplied to the machine; offsets again occur in the phase currents, giving rise to the decaying 60-Hz oscillations