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Conference Paper · August 2020

DOI: 10.1115/PVP2020-21637

4 authors, including:

Durlabh Bartaula
University of Alberta

Yong Li
University of Alberta

Samer Adeeb
University of Alberta

Proceedings of the ASME 2020
Pressure Vessel and Piping Conference
PVP2020
July 19-14, 2020, Hilton Minneapolis, Minneapolis, MN, USA

PVP2020-21637

SIMULATION OF FATIGUE CRACK GROWTH USING XFEM

Durlabh Bartaula[1], Yong Li[1] , Smitha Koduru[2], Samer Adeeb[1]

[1]University of Alberta, Edmonton, AB, Canada
[2]C-FER Technologies, Edmonton, AB, Canada

ABSTRACT

Pipelines carrying oil and gas are susceptible to fatigue failure (i.e., unstable fatigue crack propagation) due to fluctuating loading such as varying internal pressure and other external loadings. Fatigue crack growth (FCG) prediction through full-scale pipe tests can be expensive and time consuming, and experimental data is limited particularly in the face of large uncertainty involved. In contrast, numerical simulation techniques (e.g., XFEM) can be alternative to study the FCG, given that numerical models can be theoretically and/or experimentally validated with reasonable accuracy. In this study, capabilities and limitations of existing fatigue analysis code (e.g., direct cyclic approach with XFEM) in Abaqus for low cycle fatigue simulation are explored for compact-tension (CT) specimens and pipelines assuming linear elastic material behavior. The simulated FCG curve for a CT specimen is compared with that obtained from the analytical method using the stress intensity factor prescribed in ASTM E647. However, for real pipelines with elastic-plastic behavior, direct cyclic approach is not suitable, and an indirect cyclic approach is used based on the fracture energy parameters (e.g., J integral) calculated using XFEM in Abaqus. FCG law (e.g., power law relationship like Paris law) is used to generate the fatigue crack growth curve. For comparison, the FCG curve obtained through direct cyclic approach for pipelines assuming linear elastic material is also presented. The comparative studies here indicate that XFEM-based FCG simulation using appropriate techniques can be applied to pipelines for fatigue life prediction.

Keywords: Fatigue, XFEM, CT Specimen, Pipeline

NOMENCLATURE

FCG	Fatigue Crack Growth
LEFM	Linear Elastic Fracture Mechanics
EPFM	Elastic Plastic Fracture Mechanics
a	The crack size
N	Number of cycles
da/dN	Crack growth rate
K	Stress Intensity Factor (SIF)
G	Fracture Energy Release Rate
CTOD	Crack tip opening displacement
ΔK	Difference in the stress intensity factor K at maximum and minimum load
ΔJ	Difference in J-Integral at maximum and minimum load
ΔG	Difference in G at maximum and minimum load
FEM	Finite Element Method
XFEM	Extended Finite Element Method
E	Young's modulus of elasticity
PS	Plane stress
PE	Plane strain
v	Poisson's ratio

1. INTRODUCTION

Pipelines carrying oil and gas products are susceptible to fatigue failure due to varying internal pressure and other external loadings. In response to such a threat to the integrity of pipelines, it is important to predict and/or monitor the fatigue crack growth in pipelines for a good predictive maintenance strategy to prevent the occurrence of catastrophic failure due to fatigue. Fatigue damage is characterized by fatigue crack, which consists of fatigue crack nucleation, propagation and net section fracture. The nucleation phase is generally not visible and takes place at material granular level. The fatigue crack propagation phase is distinctly noticeable before net fracture takes place. This paper focuses on fatigue crack growth prediction assuming the fatigue crack has initiated using fracture mechanics-based numerical methods.

Fracture mechanics-based parameters like stress intensity factor (SIF), *J*-Integral, crack tip opening displacement (CTOD) are widely used to characterize the stable fatigue crack propagation phase in a component subjected to cyclic load. For example, the Paris law [1], a power-law form equation as shown in Eq. (1), is typically obtained by fitting the experimental data of d*a*/d*N* with respect to ΔK,

$$\frac{da}{dN} = C.\Delta K^m \tag{1}$$

where, ΔK is a parameter based on linear elastic fracture mechanics (LEFM); C and m are fatigue-related material constants.

Similar power-law form equations are used to define the crack propagation but with different fracture mechanics-based parameters. Similarly, Dowling and Begley law [2], which relates ΔJ with the crack growth rate and has form as in Eq. (2).

$$\frac{da}{dN} = C_d.\Delta J^{m_d} \tag{2}$$

where, C_d and m_d are fatigue-related material constants and obtained by fitting the experimental data as in the case of Paris law. Paris law being based on LEFM is suitable for linear elastic or brittle materials. Whereas, Dowling and Begley law is based on elastic plastic fracture mechanics (EPFM) and can be used for non-linear materials or ductile materials. Such difference mainly arises from the fact that the energy required for crack propagation in a linear elastic and non-linear elastic material is directly related to the external work done; however, in an elastic-plastic material the external work done is responsible for both plastic deformation and crack propagation.

As such, the EPFM or EPFM-based crack driving parameters such as *K*, *J*-Integral need to be determined In order to study the fatigue crack propagation in pipelines. However, calculation of crack driving parameters is a challenging task, except for solutions of SIF for standard laboratory specimens [3]. Numerical simulation (e.g., finite element methods) can be used for conditions under complex geometry and boundary conditions. Use of finite element methods to determine these parameters or simulating crack propagation together with fatigue crack growth models has been reported in [4],[5]. However, the use of conventional FEM for fatigue crack growth can be cumbersome and computationally expensive. In contrast, the use of XFEM technique, first introduced by [6], can be used for crack propagation analysis which eliminates cumbersome task of re-meshing as crack propagates. In this study, XFEM technique implemented in *Abaqus* is used to simulate the fatigue crack propagation. Existing finite element codes in Abaqus is capable of performing low cycle fatigue (LCF) analysis only when the Coffin-Manson relation is valid [7]. Due to limitation of *Abaqus* to perform LCF only, an alternative method "indirect cyclic approach" is used for performing high cycle fatigue (HCF) analysis, in which only the crack driving parameters are obtained using *Abaqus*. The fatigue analysis is applied to both a CT specimen and a pipeline section to explore the capabilities and limitations of existing fatigue analysis code using XFEM in *Abaqus*.

2. XFEM FOR FATIGUE SIMULATION

2.1 XFEM

XFEM is an extension of the conventional FEM based on the concept of the partition of the unity. The stress and displacement fields in XFEM is calculated by introducing an enrichment to the finite element which are cut by the crack. The nodes are enriched by the introduction of an additional set of *degrees of freedom (DOF)*, along with their traditional *DOF*. The displacement field within the elements is approximated by Eq. (3) and is the standard XFEM approximation [8]

$$u = \sum_{I=1}^{N} N_I(x)\left[u_I + H(x)a_{I^*} + \sum_{\alpha=1}^{4} F_a(x) b_{I^*}^{\alpha}\right] \tag{3}$$

Here, u is the displacement, $N_I(x)$ is the standard shape function; u_I is conventional nodal DOF; H(x) is the jump function across crack surface and a_{I^*} is associated DOF enriched; $F_\alpha(x)$ is the elastic asymptotic crack tip function and $b_{I^*}^{\alpha}$ is the associated nodal DOF enriched. For a propagating crack only first two terms associated with crack surface are used. Further details can be found in [6] and [7].

2.2 LCF Analysis in *Abaqus*

The built-in capability of XFEM in Abaqus performs low cycle fatigue analysis using Paris Law with some modifications. It consists of fatigue crack initiation and propagation, which are based on Eq. (4),

$$\frac{N}{c_1 \Delta G^{c_2}} \geq 1.0 \tag{4}$$

and Eq. (5), respectively,

$$\frac{da}{dN} = c_3 \Delta G^{c_4} \tag{5}$$

where, N is the number of cycles, c_1, c_2, c_3 and c_4 are fatigue properties of the material. ΔG is calculated based on virtual crack closure technique (VCCT) based on LEFM, which assumes that the strain energy released when a crack is extended equals energy required to close it. For linear elastic materials, c_3 and c_4 are related to C and m [9].Based on relation in Eq. (5), ΔN is calculated for fracturing at least one element ahead of crack tip and Δa is equal to the element length.

Instead of analyzing finite element analysis a large number of load cycles, a direct cyclic step is used in LCF. Based on direct cyclic algorithm the response at a steady state is calculated rather than at a transient state and response is extrapolated forward over number of cycles based on Coffin-Manson relation [7].

2.3 Indirect Cyclic Approach for HCF

The existing XFEM-based fatigue analysis in Abaqus is limited to LEFM and low cycle fatigue using direct cyclic method. Additionally, finite element analysis under large number of load cycles is computationally expensive. As such, an efficient and robust alternative XFEM-based fatigue analysis is desired. The basic idea here is to perform a number of static analyses to estimate the fatigue driving parameters, e.g., J-Integral, which is defined in terms of energy release rate as crack propagates [7]

$$J = \oint \left[U dy - T \left(\frac{\partial u}{\partial x} \right) ds \right] \tag{6}$$

where, U is strain energy density, T is traction on the contour integral, and u is the displacement field.

J-Integral works well as a crack driving parameter for both linear and non-linear elastic materials, but there are limitations on its use for elastic-plastic materials. In order to use J-integral as an appropriate crack driving parameter for elastic-plastic materials, several works in the past estimated ΔJ from least number of cyclic analyses. For example, [10] used Linear Matching Method to demonstrate cumulative effects over cycles, and based on their work, [11] demonstrated that if actual cyclic load is $\pm$P, then monotonic loading from 0 to +2P without unloading gives very good approximation of ΔJ.

Similar to [11], it is assumed that cyclic load from 0 to +P in this study can be replaced by a monotonic load from 0 to +P without unloading, which is equivalent to load range except that the load is asymmetrical. This approximation is justified based on Eq. (8) [12], which shows ΔJ as function of stress range, and verified later in the application example.

$$\Delta J = \oint \left[\Delta U dy - \Delta T \left(\frac{\partial u}{\partial x} \right) ds \right] \tag{7}$$

where, ΔU is change in strain energy density due to stress range, ΔT is change is traction on the contour integral due to stress range.

For application of the indirection cyclic approach to linear elastic materials using K as the fatigue driving parameter, J-Integral can be used to estimate K according to Eq. (7).

$$J = \frac{1}{8\pi} K^T . B^{-1} . K \tag{8}$$

Here, K = [K_I, K_{II}, K_{III}] are stress intensity factors along normal, shear and tangential directions and B is pre-logarithmic energy factor matrix, which is diagonal for homogeneous isotropic material [7].

In the indirect cyclic approach for HCF, XFEM model in Abaqus is used for static analysis to determine the J-Integral. The J-Integral is used to estimate the number of cycles required to extend crack by a predefined length, e.g., at least one element length ahead of crack tip according to Eq. (2). After the crack growth due to fatigue, the crack geometry is updated until a critical crack size is reached or the material toughness is reached.

Note that since the model used with the updated crack geometry is independent of the previous loading history, it is assumed that element ahead of crack tip is highly stressed but other elements are still within the elastic limit. This assumption follows that the highly stressed element is completely fractured and there is no residual stress due to stress redistribution. This is based on the assumption of fracture mechanics that the stress concentration is high at some local location and size of this area is considerably small compared to the in-plane dimension of the component. This assumption will be verified later in the application example.

3 XFEM-BASED FATIGUE ANALYSIS OF A CT SPECIMEN

3.1 CT Specimen

Before performing XFEM-based fatigue analysis of pipelines, a CT specimen [13], which allows analytical solutions, is used to explore the fatigue analysis capability in *Abaqus*. The schematic view with the geometric dimensions is shown in Figure 1. The specimen (8.25 mm thick) is pinned at the bottom clamp-hole and restrained horizontally at the top clamp-hole, where vertical load is applied.

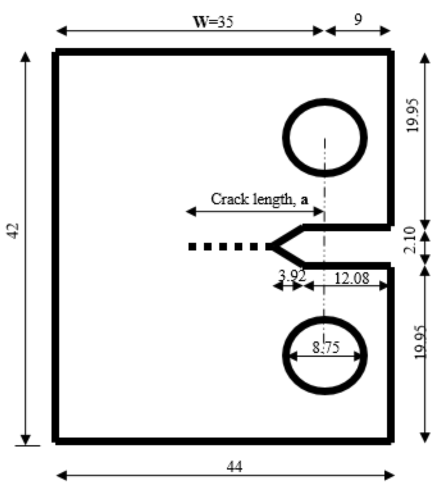

FIGURE 1: SCHEMATIC VIEW OF CT SPECIMEN. (ALL DIMENSIONS ARE IN MM AND NOT TO SCALE)

The material is API 5L X65 steel with relevant material properties [11] summarized in Figure 1 for the true stress-strain curve, Table 1 for the mechanical properties, Table 2 for the fatigue properties and Table 3 for the damage properties, respectively.

Table 1. Mechanical properties of API 5L X65 steel.

Steel Grade	Young's modulus (MPa)	Yield stress (MPa)	Ultimate stress (MPa)	Ultimate Strain (%)
API 5L X65	200000	571	709	9.1

Table 2. Fatigue properties of API 5L X65 steel.

Paris Law Coefficient, *C* (MPa and mm)	7.90E-13
Paris Law Exponent, *m*	2.76

Table 3. Damage modelling properties of API 5L X65 steel.

Maximum principal stress (MPa)	709
Critical fracture energy release rate (N/mm)	180 [16]

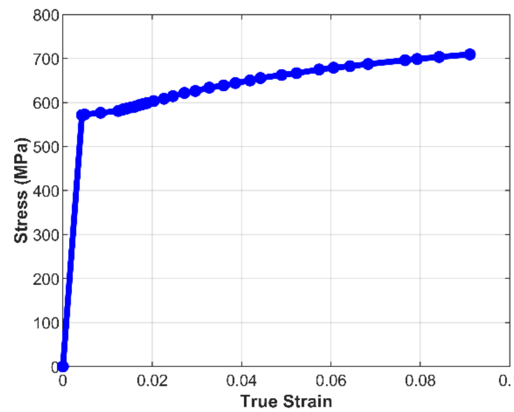

FIGURE 2: TRUE STRESS-STRAIN CURVE OBTAINED FROM UNIAXIAL TENSILE TEST FOR API 5L X65 STEEL.

3.2 XFEM-based Fatigue Analysis

3.2.1 Direct Cyclic Approach

Two dimensional (2D) models, using plane stress (CPS4) and plane strain (CPE4) elements respectively, are developed assuming linear elastic material properties for fatigue analysis using the direct cyclic approach. A cyclic load with triangular wave form having time period of 1 second and magnitude of 0 to 55 kN (42 kN), i.e. ΔP =55 kN (42 kN), is applied to the model with CPE4 (CPS4). These loads are determined such that the cyclic load in this range can cause LCF in the CT specimen based on the calculated limit load of a CT specimen with plane strain (limit load = 62 kN) and plane stress (limit load 48 kN) conditions [14]. A thickness-through initial crack of 7.125 mm is used: 7 mm is the notch length and an additional 0.125 mm is assumed to be fatigue pre-crack. This crack is introduced into the FE model with a wire element as the initial crack location, which is an input for XFEM crack modeling. In order to check the performance of the Abaqus for HCF, similar 2D models with PE and PS elements were analyzed. Where, a cyclic load with triangular wave form having time period of 1 second and magnitude of 0 to 10 kN i.e. ΔP =10 kN was applied.

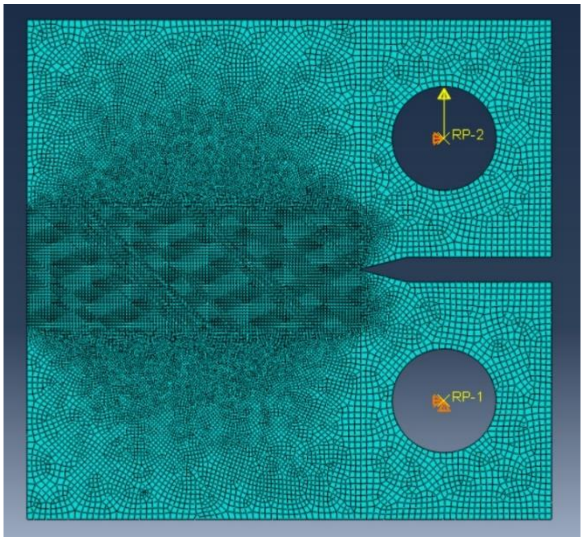

FIGURE 3: 2D FE MODEL OF CT SPECIMEN

The FCG curve generated from the Abaqus's built-in code is compared with the one obtained by numerical integration of Paris law as shown in Eq (1). In the analytical solution, the value of Δ*K* for normal mode crack opening is obtained from [3].

$$\Delta K_{\mathrm{I}} = \frac{\Delta P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \tag{7}$$

where, *ΔP* is the difference between maximum and minimum load, *B* is the width of specimen, *W* is the un-cracked ligament length, *a* is the crack length.

The FCG comparison for LCF is presented in Figure 4 and Figure 5, respectively. For CT specimen modeled with plane strain (PE) elements the FCG curve obtained from Abaqus using direct cyclic method is in reasonable agreement with the analytical solution as shown in Figure 5. However, FCG curve generated for CT specimen modeled with plane stress (PS) elements are close for first few cycles but are considerably different as loading cycle progresses and fatigue life is over predicted as shown in Figure 5 by Abaqus.

The FCG comparison obtained for HCF using Abaqus is shown in Figure 6 and it over predicts the fatigue life considerably.

$$f\left(\frac{a}{W}\right) = \frac{2+\frac{a}{W}}{(1-\frac{a}{W})^{3/2}}\left[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.60\left(\frac{a}{W}\right)^4\right] \tag{8}$$

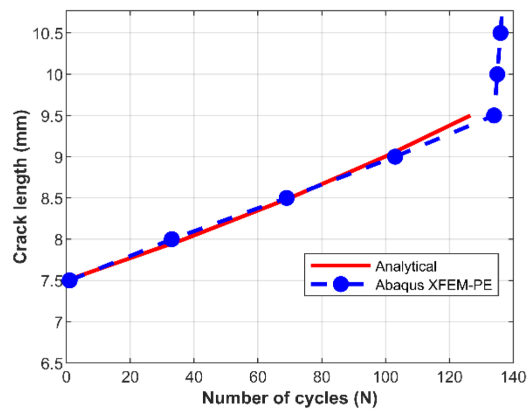

FIGURE 4: FCG CURVE FOR CT SPECIMEN WITH PLANE STRAIN (PE) ELEMENTS

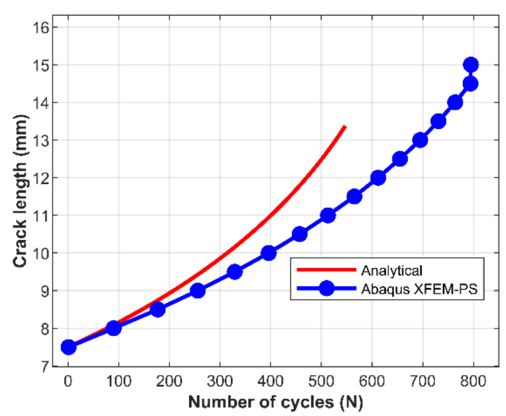

FIGURE 5: FCG CURVE FOR CT SPECIMEN WITH PLANE STRESS (PS) ELEMENTS

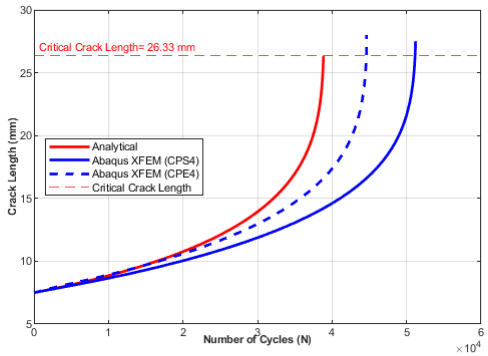

FIGURE 6: FCG CURVE FOR CT SPECIMEN WITH PLANE STRESS (PS) AND PLANE STRAIN (PE) ELEMENTS

3.2.2 Indirect Cyclic Approach

In the previous section, the built-in fatigue analysis in Abaqus was discussed; it is limited for LCF. In this section, the capability of the indirect cyclic method for HCF analysis will be studied.

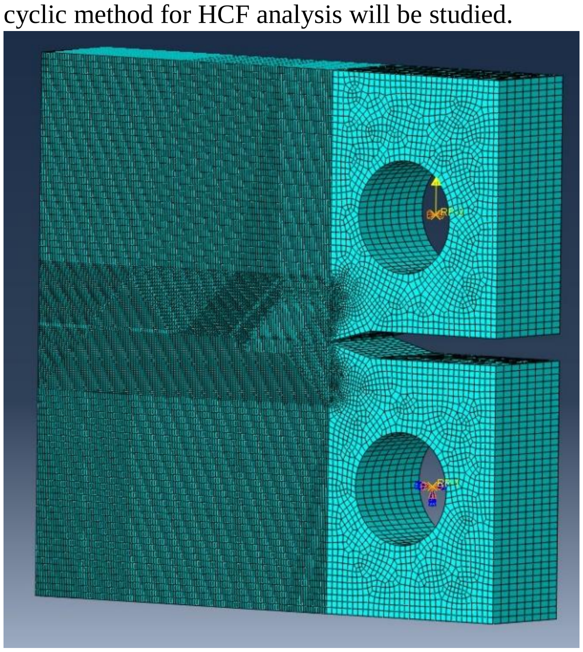

FIGURE 7: 3D FINITE ELEMENT MODEL OF CT SPECIMEN.

Since the fatigue crack driving parameters (e.g., *K*, J-Integral) are not available for 2D FE models using XFEM, a 3D FE model for the CT specimen is developed using 8-node linear brick elements (C3D8) with XFEM crack feature as shown in Figure 7. To ensure HCF in the CT specimen, a cyclic load with amplitude from 0 to +10 KN, which is very low compared to the limit load, is applied.

First, in order to verify the assumption made in section 2.3 regarding the approximate estimation of J-integral using monotonic loading (i.e., without unloading), the variation of J-Integral over a large number (e.g., 1000) of loading cycles is compared with the J-integral estimated from the same model under monotonic loading from 0 to +10 kN (see Figures 8 and 9).

As shown in Figures 8 and 9 for the value of J-Integral varies along crack front at the surface and the center, respectively, the J-integral depends on the location along the thickness direction. In general, these values are lower at the surface and higher at the center [14]. This phenomenon, as shown in Figures 8 and 9, is observed for the CT specimen with a straight crack front under both the monotonic and cyclic loading. The value of J-Integral at the surface of CT specimen is initially high but after few cycles and gets stabilized at a more or less constant value as shown in Figure 8. As it can be observed that J-Integral estimated using monotonic load is approximately equal to that estimated using cyclic loading and this difference is about 3%. The value of J-Integral at the center of CT specimen is initially low, increases slightly and gets stabilized after few cycles. The value of J-Integral estimated using monotonic load is approximately same for this case as well, the difference being about 6%. The value of J-Integral at center is greater than that at the surface, which is a reason for a well-known phenomenon of shear lip formation.

Second, Figure 10 shows the local stress-strain behavior for three elements ahead of the crack tip: element #1 right ahead of the crack tip followed by element #2 and #3. As seen in Figure 10, the element right ahead of the crack tip yields but other elements ahead are still within elastic range. This is verified for another two different crack lengths as well.

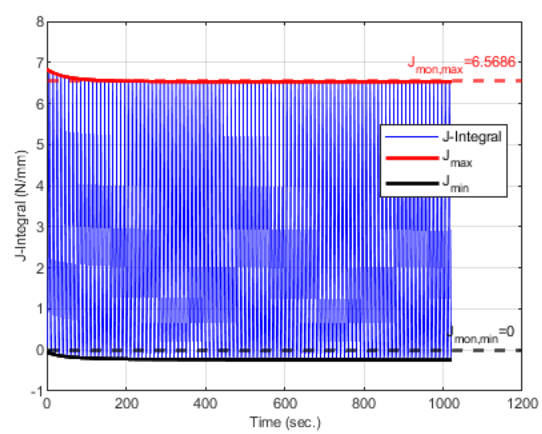

FIGURE 8: CYCLIC J-INTEGRAL AT THE SURFACE OF THE CT SPECIMEN.

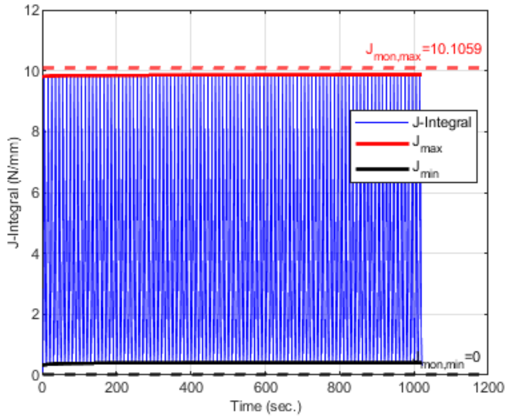

FIGURE 9: CYCLIC J-INTEGRAL AT THE CENTER OF THE CT SPECIMEN.

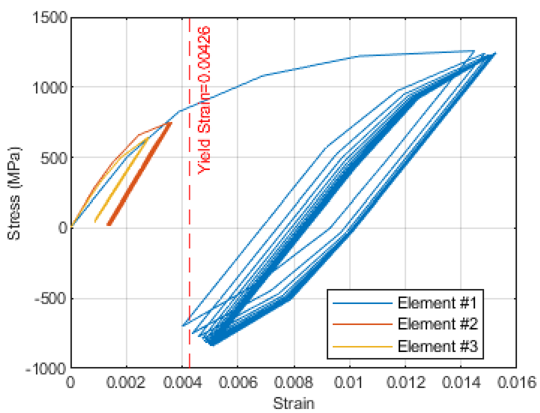

FIGURE 10: CYCLIC STRESS STRAIN CURVE OF ELEMENTS AHEAD OF THE CRACK TIP LOCATION OF CT SPECIMEN

The XFEM-based HCF crack growth curve is compared with the analytical solution for linear elastic materials for validation purpose here. The J-integral or equivalent *K* obtained from the XFEM model under monotonic loading, along with other fatigue properties provided in Table 2, is used to generate FCG curve (see Figure 11) using the indirect cyclic method. Note that ΔK is calculated using the average values of the K at the center and at the surface of the CT specimen.

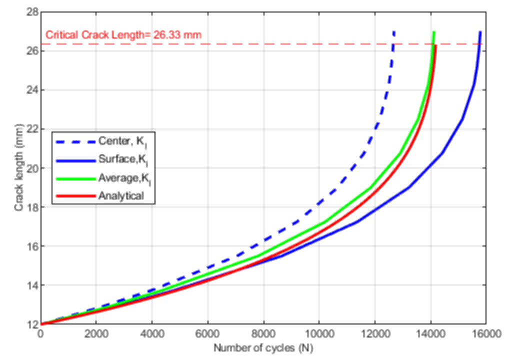

FIGURE 11: FCG CURVE FOR CT SPECIMEN USING INDIRECT CYCLIC METHOD

4 XFEM-based Fatigue Analysis of a Pipeline Segment

4.1 Pipeline Segment

A pipeline segment with the same material but slightly different lengths from the pipeline specimen in [14] is studied. The fatigue properties are provided in Table 4. The outer diameter of pipe is 762 mm; the wall thickness is 8.5 mm; and the total length of pipeline is 1500 mm, as shown in Figure 12. The pipeline segment is constrained at both ends and loaded on the internal surface with internal pressure.

Table 4. Fatigue properties of API 5L X65 used in pipeline section.

Fatigue Coefficient, C_d (N-mm)	1.63e-05
Fatigue Exponent, m_d	1.38

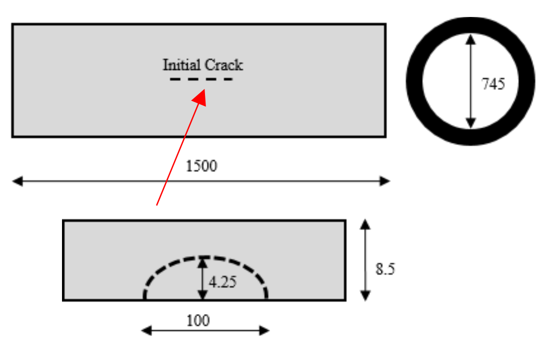

FIGURE 12: SCHEMATIC DIAGRAM OF THE PIPELINE SEGMENT WITH CRACK

4.2 XFEM-based Fatigue Analysis

4.2.1 Direct Cyclic Approach

A 3D finite element model, as shown in Figures 13 and 14, is developed for the pipeline segment considered. Two types of elements are used: 8-node linear brick elements (C3D8) for the domain containing the crack, and 4-node linear shell elements (S4) for rest of the pipe body. An elliptical initial surface crack along the longitudinal direction is introduced into the model as an input for XFEM crack modeling. Major and minor radii of the crack are 100 mm and 4.25 mm, respectively, as shown in Figure 12. Linear elastic material properties is used due to the limitation of the direct cyclic approach. A triangular wave form cyclic pressure, varying from 0 to 6 MPa with time period of 1 second, is applied on the internal surface of the pipe.

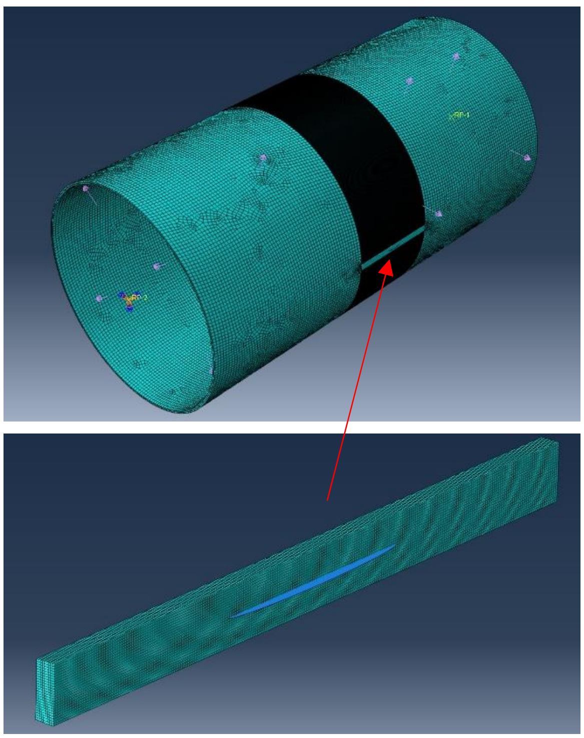

Figure 14: 3D FE MODEL OF PIPELINE SEGMENT WITH CLOSE UP VIEW OF MESH IN PIPELINE SECTIONS EXPECTED CRACK PROPAGATION ZONE; MESH SIZE OF 1 MM X 1MM

FCG curves obtained from the direct cyclic approach for pipeline section are shown in Figures 15 and 16. Figure 15 is for the crack depth along the wall thickness direction (i.e., elliptical crack minor radius), and Figure 16 is for the crack length along the longitudinal direction (i.e., elliptical crack major radius). Fatigue crack growth rate is relatively higher along the wall thickness direction compared to the longitudinal direction.

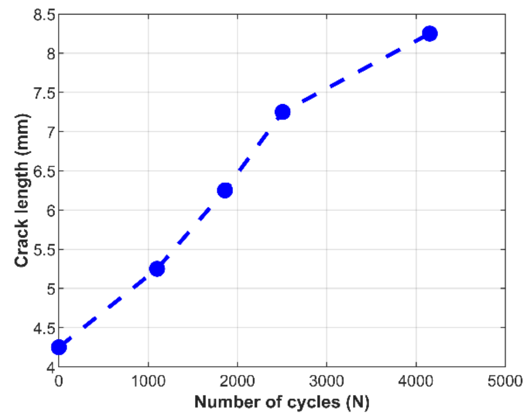

FIGURE 16: FCG CURVE FOR PIPELINE SECTION; NUMBER OF LOADING CYCLES VERSUS MINOR RADIUS OF ELLIPTICAL CRACK FRONT.

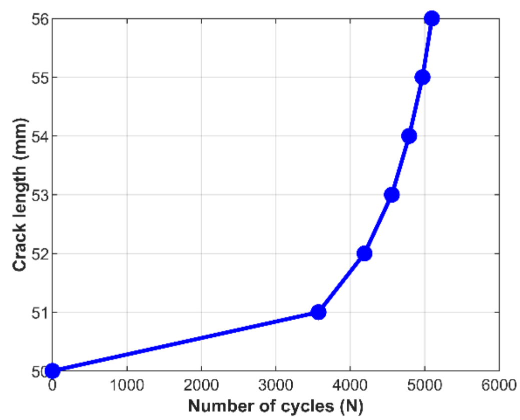

FIGURE 17: FCG CURVE FOR PIPELINE SECTION; NUMBER OF LOADING CYCLES VERSUS MAJOR RADIUS OF ELLIPTICAL CRACK FRONT

4.2.2 Indirect Cyclic Approach

The indirect cyclic approach is applied to the pipeline studied above except that elastic-plastic material properties are considered, which is the relative advantage of this approach compared with the direct cyclic approach. FCG curves for pipeline segment with an elliptical crack front is shown in Figure 18 and 19. Figure 18 is for the crack depth along the wall thickness direction (i.e., elliptical crack minor radius), and Figure 19 is for the crack length along the longitudinal direction (i.e., elliptical crack major radius). Fatigue crack growth rate is relatively higher along the wall thickness direction compared to the longitudinal direction. The computational time using this method was comparatively lower to the direct cyclic approach. However, the indirect cyclic approach needs to be validated with the experimental data and this is the undergoing work of the authors.

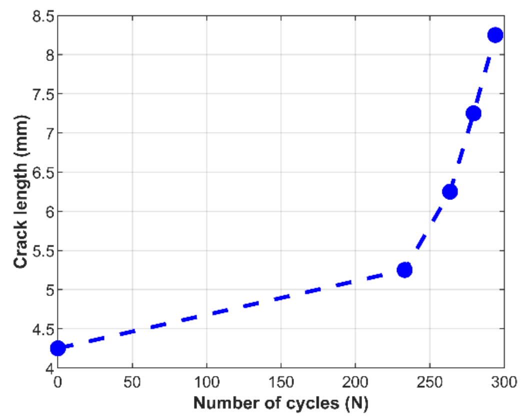

FIGURE 18: FCG CURVE FOR PIPELINE SECTION USING PROPOSED METHOD; NUMBER OF LOADING CYCLES VERSUS MINOR RADIUS OF ELLIPTICAL CRACK FRONT.

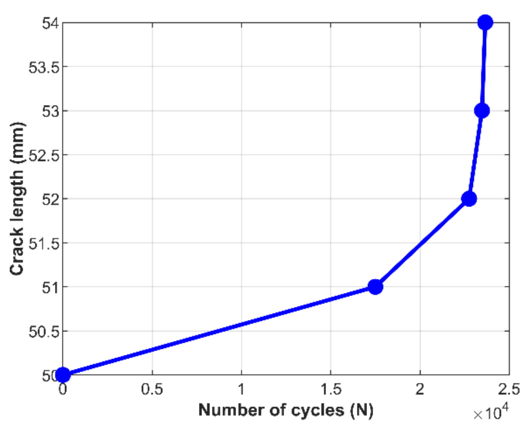

FIGURE 19: FCG CURVE FOR PIPELINE SECTION USING PROPOSED METHOD; NUMBER OF LOADING CYCLES VERSUS MAJOR RADIUS OF ELLIPTICAL CRACK FRONT.

5 CONCLUSION

The XFEM-based fatigue analysis in Abaqus was studied in this paper, including the built-in method (i.e., direct cyclic method) and the indirect cyclic method. It was found that the built-in method is limited for LCF analysis of a CT specimen with linear elastic materials. However, the fatigue result of a CT specimen obtained using indirect cyclic approach for HCF shows reasonable agreement with the analytical solution available in the literature. These two fatigue analysis methods are applied to a pipeline segment as well. The preliminary study in this paper shows the potential of the indirect cyclic method for fatigue analysis of pipelines with elastic-plastic materials. The validation of the method is to be further validated with experimental data from the literature in the near future.

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