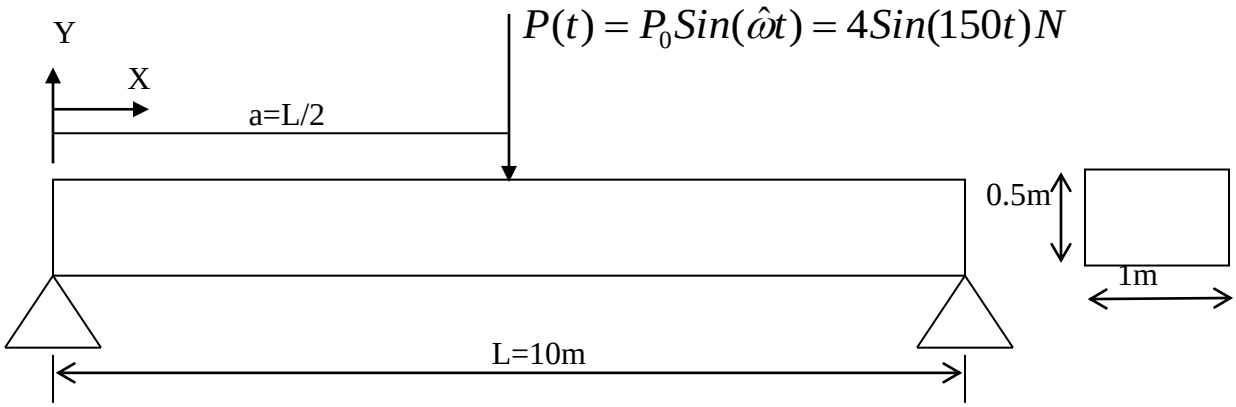
HW# 4: 3-D Benchmark Problem Statement (Due March 3, 2022)



- Use your fem3d-dynamic code with 20 node brick elements to obtain the mid-span displacement of the 10 m long simply- supported steel beam shown above.
- The beam is initially at rest. Use a 10x1x1 mesh and 3x3x3 integration.
- Use uniform time step size of 0.001s for a total analysis time of at least 0.1s.
- Material properties for steel are Young's Modulus E= 200 GPa, Poisson's Ratio= 0.3, and density = 7800 Kg/m^3, A= area of beam cross section, and I= area moment of inertia of beam cross section about the mid-plane.
- The exact solution for displacement history at beam mid-span can be obtained using the given equation. Compare (tabulate and plot) exact solution with the finite element solution for model verification over at least 0.1 s, and find % error. Here $\hat{\omega}$ is the frequency of the applied load and is equal to 150 rad/sec.

Exact Solution for beam deflection w(x,t) is given by,

w(x,t)=

$$\frac{2P_o L^3}{\pi^4 EI}\left\{\frac{\sin\left(\frac{\pi a}{L}\right)\sin\left(\frac{\pi x}{L}\right)}{1-\left(\frac{\hat{\omega}}{\hat{\omega}_1}\right)^2}\left[\sin(\hat{\omega}t)-\frac{\hat{\omega}}{\hat{\omega}_1}\sin(\hat{\omega}_1 t)\right]+\frac{\sin\left(\frac{2\pi a}{L}\right)\sin\left(\frac{2\pi x}{L}\right)}{2^4-\left(\frac{\hat{\omega}}{\hat{\omega}_1}\right)^2}\left[\sin(\hat{\omega}t)-\frac{\hat{\omega}}{2^2\hat{\omega}_1}\sin(\hat{\omega}_1 t)\right]+.....\right.$$

Where,

$$\hat{\omega}_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EIg}{\gamma\Omega}}$$

$$\frac{\gamma\Omega}{g} = \rho A$$