SOFTWARE SIMULATION OF THE EEG

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ABSTRACT

The literature contains many examples of digital procedures for the analytical treatment of electroencephalograms, but there is as yet no standard by which those techniques may be judged or compared. This paper proposes one method of generating an EEG, based on a computer program for Zetterberg's simulation. It is assumed that the statistical properties of an EEG may be represented by stationary processes having rational transfer functions and achieved by a system of software filters and random number generators.

The model represents neither the neurological mechanism response for generating the EEG, nor any particular type of EEG record; transient phenomena such as spikes, sharp waves and alpha bursts also are excluded. The basis of the program is a valid 'partial' statistical description of the EEG; that description is then used to produce a digital representation of a signal which, if plotted sequentially, might or might not by chance resemble an EEG, that is unimportant. What is important is that the statistical properties of the series remain those of a real EEG; it is in this sense that the output is a simulation of the EEG. There is considerable flexibility in the form of the output, i.e. its alpha, beta and delta content, which may be selected by the user, the same selected parameters always producing the same statistical output. The filtered outputs from the random number sequences may be scaled to provide realistic power distributions in the accepted EEG frequency bands and then summed to create a digital output signal, the 'stationary EEG'. It is suggested that the simulator might act as a test input to digital analytical techniques for the EEG, a simulator which would enable at least a substantial part of those techniques to be compared and assessed in an objective manner. The equations necessary to implement the model are given. The program has been run on a DEC1090 computer but is suitable for any microcomputer having more than 32 kBytes of memory; the execution time required to generate a 25 s simulated EEG is in the region of 15 s.

Keywords: Electroencephalographs, computer, model

INTRODUCTION

Computer analyses of the electroencephalogram (EEG) aim to extract information from the signal and present it in a more objective and convenient form for interpretation. Many analytical techniques; maximum likelihood[1], linear prediction, maximum entropy, Kalman filtering[2,3] and others have been developed. They use only a few parameters to describe the signal but however efficient that description may be, there remains the difficult problem of convincing medical colleagues of its validity and utility. For this purpose, the techniques are frequently applied to simulated data, sine waves, white noise etc, whose parameters are known and hence valid comparisons may be made among different analytical techniques. Various methods for simulating the EEG have been suggested: autoregressive series[4], analogue techniques[5–9] and others. An autoregressive series may be used to simulate an EEG by filtering white noise through an autoregressive process having the desired characteristics. Analogue techniques[5] in general are electronic circuits in which independent white noise signals, generated by devices such as zener diodes, are filtered by active low-pass and band-pass filters having the prescribed centre frequencies and bandwidths; the filtered outputs are scaled and summed to obtain the required analogue EEG simulation. In order to evaluate discrete analytical techniques the analogue EEG signal must be converted to a digital form; it is therefore desirable to generate the EEG data directly in that form. This paper deals with such a method and in principle is based on the analogue technique of Zetterberg[5].

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0141–5425/85/040275–07 $03.00

In the sections which follow, the first part describes a model on which the simulation is based. The second deals with the implementation of the simulation using finite impulse response filtering by means of fast convolution. The third illustrates the method of simulation by an example and also deals with the practical problems and results obtained.

THE BASIC MODEL

An EEG may be regarded as a statistical process with two components; (1) a stochastic component which is stationary over short epochs and (2) transient components (wave train, spikes and sharpwaves) that arise sporadically. The transients can be considered to be superimposed on the stationary stochastic component, which is referred to as the 'background activity' and is our present concern[3,12].

The spectral properties of a stationary stochastic process can be represented by a model consisting of a white noise-generator and a filter; this model is attractive in this application because we may reasonably assume that EEG signals are formed by an organic system which 'sums and filters' the primary impulses. A technical reason for its adaption is that filtering is a flexible way of shaping a spectrum; moreover, filter functions are well established and readily described in parametric form. It should be emphasized that the model is not claimed to imitate the complicated neurological process of generating an EEG, but represents a description of the EEG signal[1].

It is assumed that the spectral density function (SDF) of the EEG signal can be described by a rational function of the form[5]

$$\frac{Q(f^2)}{P(f^2)} \quad (1)$$

where $P(x)$ and $Q(x)$ are polynomials in x with degree p and q respectively.

Equation (1) allows resonances to appear in the SDF and it is possible to control the resonance frequencies, the band-width of resonance peaks, and the power content related to those peaks. On partial fraction expansion, (1) can be expressed as a sum of spectral components each corresponding to one term in the partial fraction expansion. These terms are of two types, depending upon where the poles or resonances are located. With type 'I', the pole is located on the negative real axis in the S–plane. ($S = \sigma + j\omega$). Whereas for type 'II', poles appear in complex conjugate pairs with negative real part. That is, the SDF of two functions are:

$$\text{type 'I': } R_{Ii}(f) = \frac{C}{f^2 + \sigma_i^2} \quad (2)$$

$$\text{type 'II': } R_{IIi}(f) = \frac{Af^2 + B}{(f_i^2 + \sigma_i^2 - f^2)^2 + (2f\sigma_i)^2} \quad (3)$$

the first function has a resonance peak at $f = 0$ and the second has a peak close to $\pm f_i$ when σ_i is much less than f_i. The parameter σ_i measures the bandwidth of resonance peak for type 'I' and the half bandwidth for type 'II'. A weighted sum of these two will give the function (1). The correlation functions corresponding to (2) and (3) are given by

$$r_{Ii}(\tau) = G_i\, e^{(-2\pi\sigma_i|\tau|)} \quad (4)$$

$$r_{IIi}(\tau) = (G_i \cos 2\pi f_i \tau - H_i \sin 2\pi f_i |\tau|)\, e^{(-2\pi\sigma_i|\tau|)} \quad (5)$$

The parameter G_i expresses the power contribution from each spectral component, whereas H_i gives a measure of asymmetry for the SDF relative to the resonance frequency f_i. The frequency parameters σ_i and f_i agree in two sets of expressions, whereas relations between G and H parameters A and B are involved.

In effect we can describe the spectral properties of an EEG by using certain frequency and power parameters; the δ activity is described by type 'I' function and θ, α and β activities are described by type 'II' functions[5]. In general, for complete description of the SDF, one more type of function should be added in order to take into account an activity that is due to constant spectral density within the frequency band under consideration. The corresponding power is denoted by E. This activity corresponds to white noise and results in a component of type '0'. In summary, the model describing the EEG must have the following parameters[1]:

For type '0' : Power parameter E.
type 'I' : Frequency parameter σ_i and power parameters G_i.
type 'II': Frequency parameter σ_i and f_i, power parameters G_i and H_i.

IMPLEMENTATION

Based on the model described above, and knowing the frequency and power parameters of different activities involved, simulation can be carried out by the procedure described in this section.

Delta activity can be described by type 'I' function; the transfer function which can realize this is:

$$\text{in analogue domain, } T_\delta(s) = \frac{1}{s + 2\pi\sigma_\delta} \quad (6a)$$

and

$$\text{in discrete domain, } T_\delta(z) = \frac{1}{1 - \exp(-2\pi\sigma_\delta T_s)\, z^{-1}} \quad (6b)$$

where σ_δ is the −3 dB cut-off frequency in Hz and $(1/T_s)$ is the sampling frequency.

The θ, α and β activities are described by type 'II' function and the transfer function realizing this is

$$T_i(s) = \frac{s + 2\pi\sigma_{zi}}{(s + 2\pi\sigma_i)^2 + (2\pi f_{0i})^2} \quad (7a)$$

and

$$T_i(z) = \frac{1 - \exp(-2\pi\sigma_{zi}T_s)\, z^{-1}}{1 - 2\exp(-2\pi\sigma_i T_s)\cos(2\pi f_{0i} T_s)\, z^{-1} + \exp(-4\pi\sigma_i T_s) z^{-2}} \quad (7b)$$

in the analogue and discrete domains respectively. σ_{zi}, σ_i and f_{0i} are different for θ, α and β activities, σ_{zi}, is the −3 db zero frequency. Simulation of EEG is carried out by filtering independent random number sequences by filter transfer functions corresponding to different activities and adding the filter outputs with appropriate gain factors to get the required power distribution among individual activities. This scheme is shown in *Figure 1*.

Independent random number sequences having uniform distribution function are generated by changing the seed value. The impulse responses of filters corresponding to different activities are found by taking the inverse transform of the transfer function describing the activity.

The δ–activity transfer function in the analogue domain (6a) and the discrete domain (6b) has the impulse response given respectively by

$$y(t) = \exp(-2\pi\sigma_\delta t) \quad (8a)$$

and

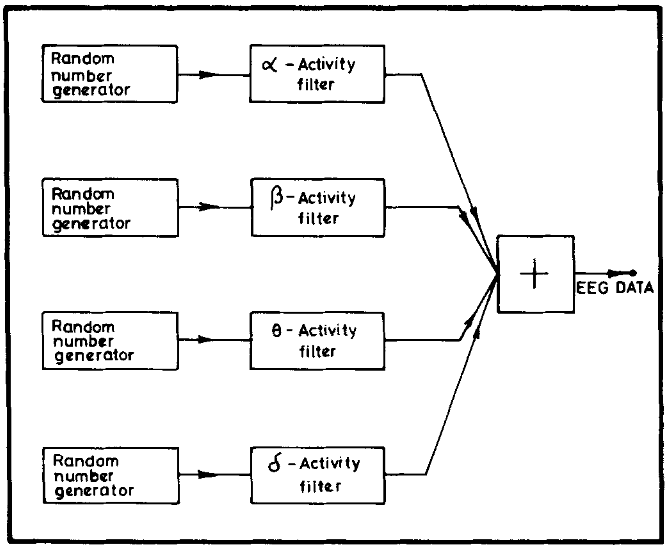


Figure 1 Schematic diagram of the EEG simulator

$$y(n) = x(n) - ay(n-1) \qquad (8b)$$

where $a = -\exp(-2\pi\sigma_\delta T_s)$, $x(n) = \delta(n)$

The impulse response in analogue and discrete domains for α, θ and β activities are obtained from Equations (7a) and (7b) respectively as

$$y(t) = \frac{[(\sigma_{zi} - \sigma_i)^2 + f_{0i}^2]^{1/2}}{f_{0i}} \exp(-2\pi\sigma_i t)$$

$$\sin(2\pi f_{0i} t + \psi) \qquad (9a)$$

where

$$\psi = \tan^{-1}\left(\frac{f_{0i}}{\sigma_{zi} - \sigma_i}\right)$$

and

$$y(n) = x(n) + b_1\, x(n-1) - \sum_{k=1}^{2} a_k y(n-k) \qquad (9b)$$

where

$x(n) = \delta(n)$,

$b_1 = -\exp(-2\pi\sigma_{zi} T_s)$

$a_1 = -2\exp(-2\pi\sigma_i T_s)$

and

$a_2 = \exp(-4\pi\sigma_i T_s)$

Impulse responses are sampled by taking into account the maximum frequency content of the EEG signal; to avoid the leakage effect the sampled impulse response values are further truncated and subjected to a Hamming window. For each activity, a random number sequence is filtered by the corresponding filter transfer function. This filtering is done by convolution using the overlap add method[10].

If each random number sequence has N numbers, it is divided into NS segments of length L. Then, if M is the length of the impulse response coefficients to avoid time domain aliasing, the length of the required discrete Fourier transform (DFT) is $(L + M - 1)$. For each random number sequence, the convolution is executed by multiplying the DFT of the random number sequence segment by the DFT of the impulse response (each of length $(L + M - 1)$) and taking the inverse DFT of the computed product. These NS sets of filtered outputs corresponding to each segment are stored in a matrix array of size $(NS \times L)$.

In the matrix, the data of the first row starting from $(L + 1) - (L + M - 1)$ i.e., $(M = 1)$ points overlap over the points starting from 1st – $(M - 1)$th point of the second row. Similarly, the second row data points overlap over the third and so on. According to overlap add method, the data corresponding to 1st – $(M = 1)$th points of the second row are added to $(L + 1)$th – $(L + M - 1)$th points of the first row. The third row data points from 1st – $(M = 1)$th are added to $(L + 1)$th – $(L + M - 1)$th points of the second row. This repeats for the following rows. The data points of the 1st – $(M - 1)$th points of the second, third, fourth etc., rows are made zero and further 1st – $(M - 1)$th points of the second, third, fourth etc., are deleted. The data points in different rows are put in a column matrix of dimension $(N + M - 1)$. This column matrix gives the filtered output corresponding to one activity. The sum of the squares of this filtered data is also found. i.e.,

$$\sum_{k=1}^{N+M-1} \Phi_i^2(k)$$

where $\Phi_i(k)$ is the filtered output for the ith activity ($i = \alpha$, β, δ and θ) and kth sample. The same procedure is carried out for all activities. The different filtered outputs are stored in different arrays.

The required power distribution is done as explained below:

$$\text{If} \quad \sum_{k=1}^{N+M-1} \Phi_\alpha^2(k) = X,$$

$$\sum_{k=1}^{N+M-1} \Phi_\beta^2(k) = Y$$

and

$$\sum_{k=1}^{N+M-1} \Phi_\delta^2(k) = Z \qquad (10)$$

(Assuming that only α, β and δ activities are present). If the percentage of power distribution of each activity in the process is $P{:}Q{:}R$, then

$$P = \frac{G_1^2 \cdot X \cdot 100}{X'+Y'+Z'}, \qquad Q = \frac{G_2^2 \cdot Y \cdot 100}{X'+Y'+Z'}$$

and

$$R = \frac{G_3^2 \cdot Z \cdot 100}{X'+Y'+Z'} \qquad (11)$$

where $G_2^2 = (Z/Y)\cdot(Q/R)\cdot G_3^2$, $G_1^2 = (P/R)\cdot(Z/X)\cdot G_3^2$, $X' = G_1^2 X$, $Y' = G_2^2 Y$ and $Z' = G_3^2 Z$.

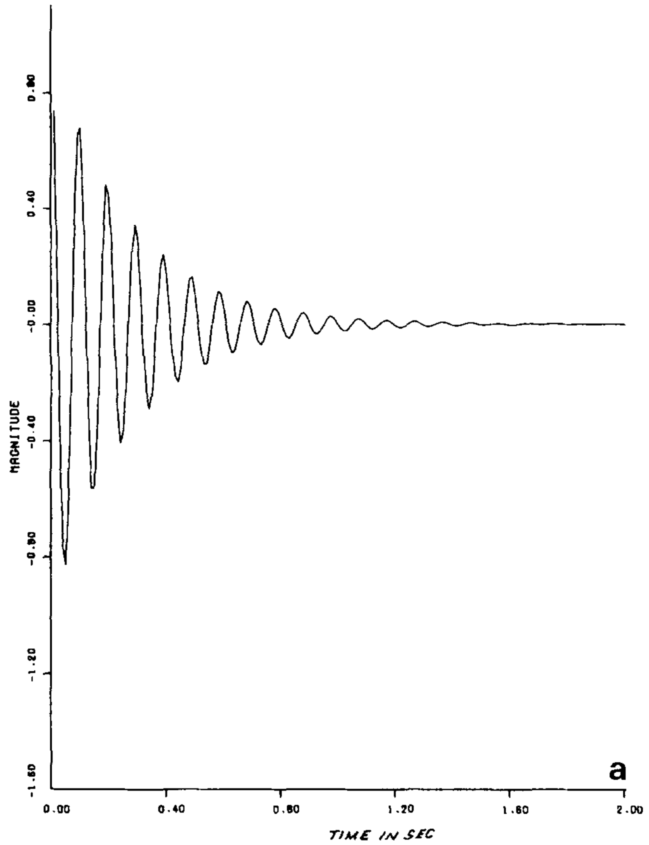
Assuming G_3^2, (say = 10) we can find G_1^2 and G_2^2 and further G_1, G_2 and G_3 which are the required gain factors to obtain the desired power distribution. The filtered sequences for α, β and δ are multiplied by G_1, G_2 and G_3 respectively and then summed to obtain the required EEG data. This method has the advantage that it allows independent control of frequency and power parameters, but it has the limitation that all possible values of H_i cannot be realized with a second order network; in practice, this is not a serious obstacle.

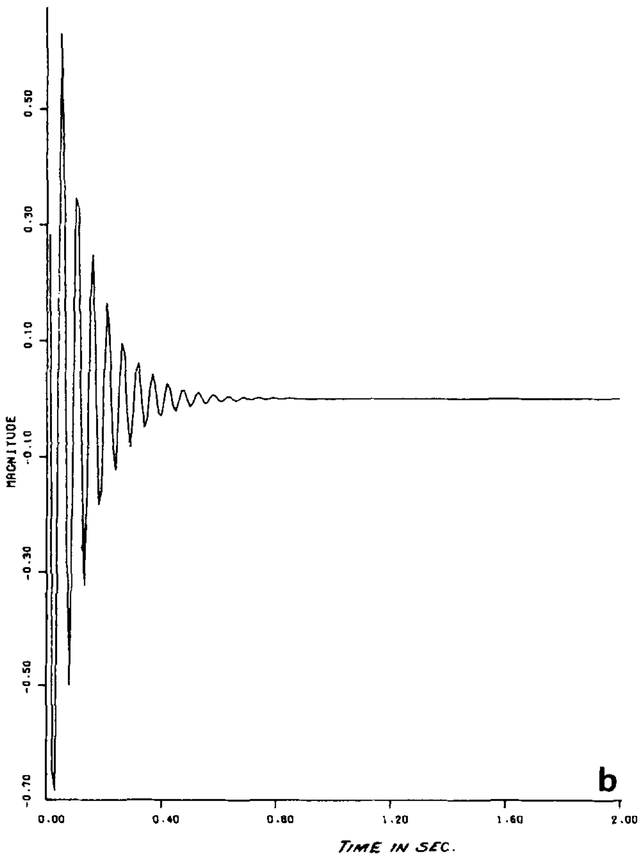
EXAMPLE AND RESULTS

To illustrate the simulation and its performance, the following parameters are chosen for different activities in a hypothetical EEG, they are the same as those estimated by Zetterberg[5] and are shown in *Table 1*. H_i are assumed to be small compared with

Table 1 Parameters of EEG registration

Activity	σ_i(Hz)	f_{0i}(Hz)	G_i(%)
α	0.58 ± 0.03	10.25 ± 0.03	63 ± 11
β	1.36 ± 0.1	18.9 ± 0.1	4 ± 0.9
δ	1.27 ± 0.07	0.0	33 ± 6





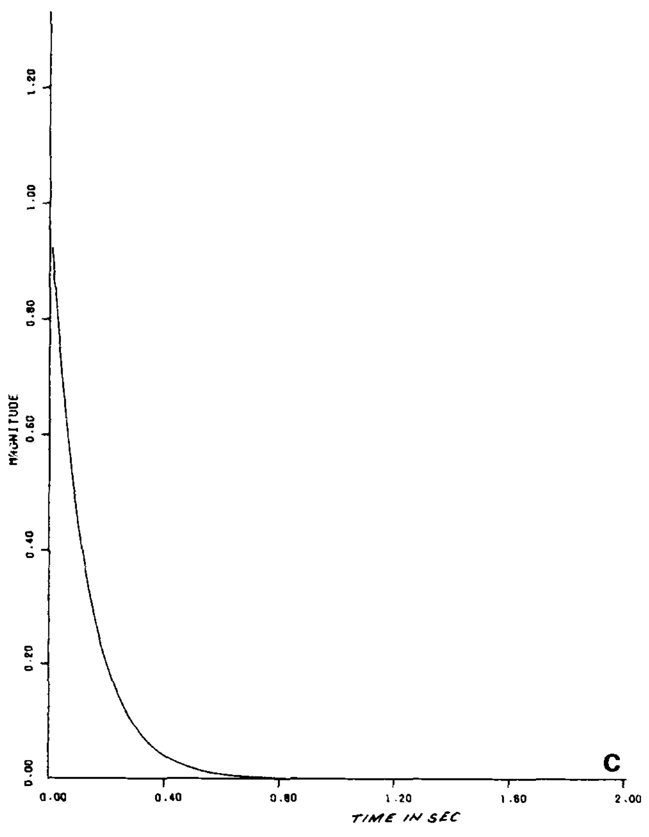


Figure 2 Truncated impulse responses of filters that realize different activities (a) α-activity, (b) β-activity and (c) δ-activity

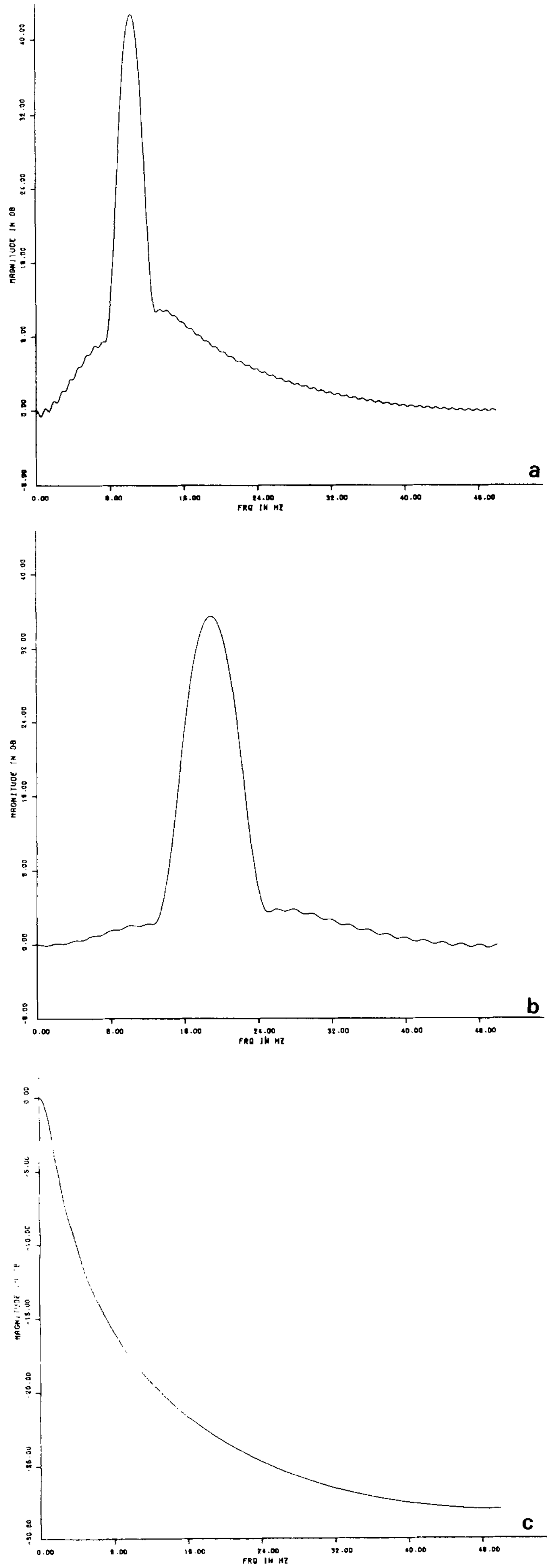


Figure 3 Log-magnitude frequency spectrum of the impulse responses (shown in *Figure 2*) for different activities a) α-activity, (b) β-activity and (c) δ-activity

Table 2 Frequency characteristics of filter impulse responses

Activity	σ_i(Hz) Gain (dB)	f_{0i}(Hz)
α	0.586 (−2.57)[a] 0.596 (−2.56)[b]	10.35
β	1.465 (−2.86)[a] 1.367 (−2.82)[b]	19.043
δ	1.269 (−2.67) 1.367 (−3.00)	0.0

[a] and [b] refer to the half bandwidths on the upper and lower sides of f_{0i}

G_i for α and β activities, the tolerances given indicating deviations which one finds if data are analysed repeatedly for non-overlapping epochs.

Using these values for σ_i and f_{0i} the impulse responses are found using (8a) and (9a). The impulse responses are sampled at 100 Hz as most of the EEG activity is confined to frequencies below 40 Hz; at this sample rate the sampling theorem is also satisfied. The impulse responses for α and δ are truncated to 1.13 s, corresponding to decays of 97.28% and 99.986% respectively. For β–activity the impulse response is truncated to 0.5 s which corresponds to a decay of 99.7%. The impulse responses are shown in *Figure 2*; to each truncated impulse response a Hamming window is applied. The log-magnitude spectrum of each sequence is found and is shown in *Figure 3*, from which it will be seen that the truncated Hamming windowed impulse responses have the parameters given in *Table 2*. N, the random number sequence length, = 4400; the impulse response length $M = 113$ for α and δ activity and 50 for β-activity; and for the random number sequence segment length $L = 400$. The random number sequences are generated with seed values 65549 for α, 85565 for β and 40083 for δ.

For an efficient convolution computation (a filtering operation), given the impulse response length, the data length can be chosen as suggested by Brigham[11]. It has been found that the truncation of a random number sequence without removing its mean, results in a steep fall at 0.0 Hz from 0 dB to −22 dB over a range of 0 to 2 Hz; this is due to convolution of the Fourier transform of the window function with that of the random number sequence. As a result, when a segment of random number sequence is passed through the δ filter, the output will not have exactly desired characteristics, even though the DFT of the impulse response for δ–activity satisfies the required specifications (i.e., −3 dB bandwidth of 1.27 Hz); the filtered output will instead exhibit a 12.39 dB attenuation at 1.27 Hz. In order to overcome this, in the DFT of the windowed random number sequence the points over which steep fall occurs are replaced by the values at some other region in the spectrum where it is almost flat; the DFT of the windowed random number sequence is modified whenever it is used for further processing. With this modification it has been found that the output for the −3 dB δ

Table 3 Frequency parameters for individual activities

Parameter (Hz)	α-activity	β-activity	δ-activity
σ_i	0.781[a] (−2.929 dB)	1.953[a] (−2.83 dB)	
	0.781[b] (−3.248 dB)	2.051[b] (−3.08 dB)	1.07 (−3.0 dB)
f_{0i}	10.25391	19.14063	0.0

[a] and [b] refer to *Table 2*

bandwidth satisfies more closely the original specification. The spectrum of α and β activities are also affected in a similar way by the windowing of the random number sequence and exhibit peaks at 0.0Hz. The modified spectrum of random number sequence also removes the unwanted peaks and improves the spectrum for α and β activities.

Table 4 Frequency parameters for the simulated EEG activity

Activity	σ_i(Hz)	f_{0i}(Hz)
α	0.781 (−2.92 dB)[a] 0.781 (−3.3 dB)[b]	10.2539
β	.... spread	
δ	1.07 (−2.99 dB)	0.0

[a] and [b] refer to *Table 2*

To determine the accuracy of the frequency parameters of the simulated data, the SDF of each activity (α, β and δ) separately and that of the EEG (α, β and δ together) are computed using the Welch method[10], and are shown in *Figure 4*. In

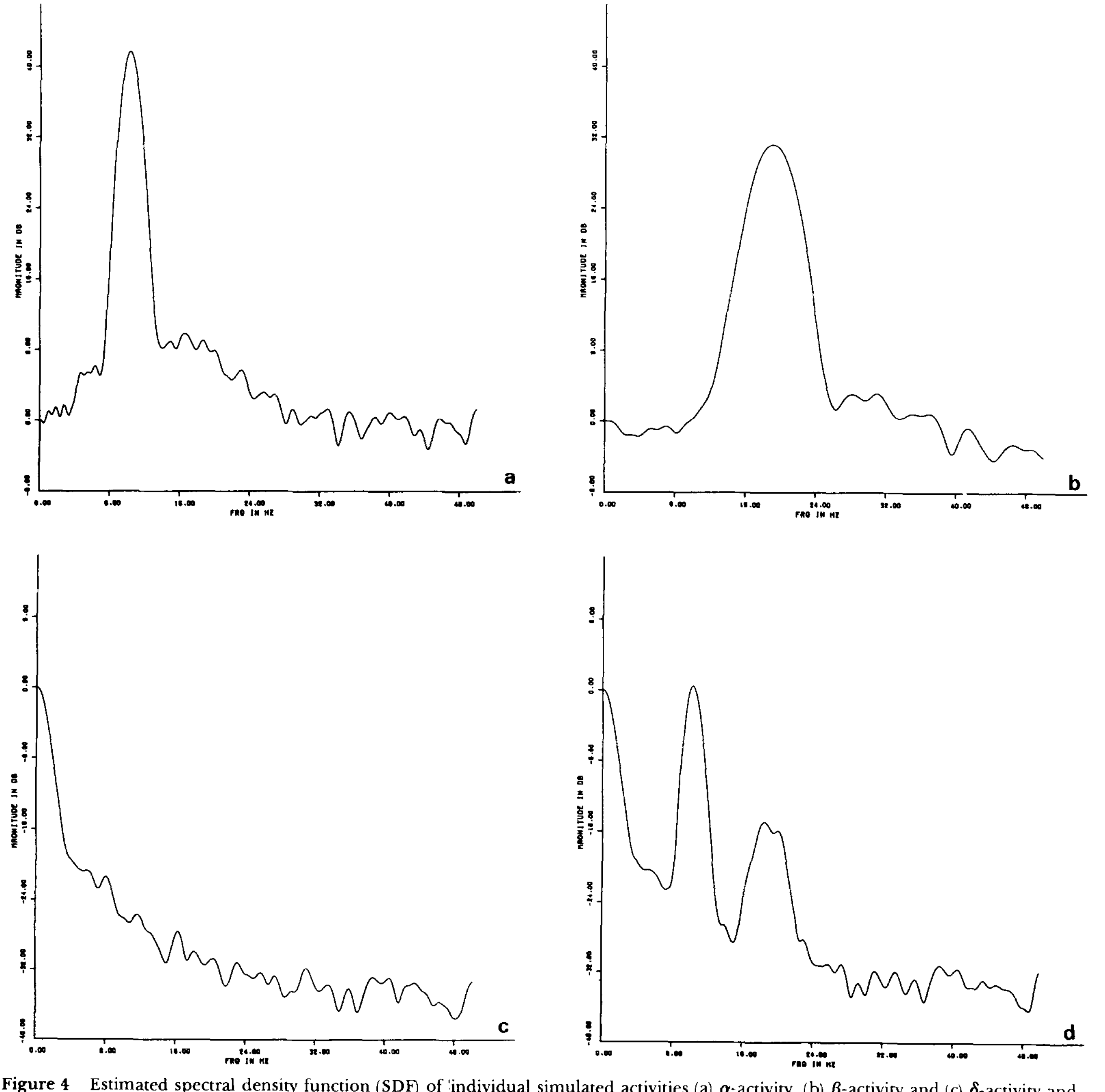


Figure 4 Estimated spectral density function (SDF) of individual simulated activities (a) α-activity, (b) β-activity and (c) δ-activity and (d) that of simulated EEG data

computing the SDF for α, β and δ separately the lengths chosen are 113, 50 and 113 points respectively, which were used to compute the spectrum of the impulse responses of different activities. In estimating the SDF of the EEG data, the length of the data is 113 points, which ensures the best resolution for α–activity. In each case, the data is Hamming windowed and padded with zeros to a length to 1024 points, which provides a resolution of about 0.1 Hz (interpolation); the number of spectra averaged is 25. The frequency parameters of the simulated individual activities and that of EEG data are given in *Tables 3* and *4*

The percentage of power distribution for α, β and δ is found by computing

$$P_\alpha = \frac{\sum_{k=1}^{N+M-1} G_1^2 \Phi_\alpha^2(k)}{\sum_{k=1}^{N+M-1} [G_1 \Phi_\alpha(k) + G_2 \Phi_\beta(k) + G_3 \Phi_\delta(k)]^2},$$

$$P_\beta = \frac{\sum_{k=1}^{N+M-1} G_2^2 \Phi_\beta^2(k)}{\sum_{k=1}^{N+M-1} [G_1 \Phi_\alpha(k) + G_2 \Phi_\beta(k) + G_3 \Phi_\delta(k)]^2}$$

and

$$P_\delta = \frac{\sum_{k=1}^{N+M-1} G_3^2 \Phi_\delta^2(k)}{\sum_{k=1}^{N+M-1} [G_1 \Phi_\alpha(k) + G_2 \Phi_\beta(k) + G_3 \Phi_\delta(k)]^2}$$

where $N + M - 1 = 4512$.

P_α, P_β and P_δ computed for the simulated data are 62.38%, 3.96% and 32.67% respectively.

COMPUTATIONAL PROCEDURE

1. Compute the impulse response for the various activites – for δ–activity using equation (8a), for all other activities using equation (9a).
2. Generate an independent random number sequence for each activity.
3. Filter each of the random number sequences by a filter impulse response obtained in step 1.
4. For a fixed length of each of the filtered sequences, compute the power using equation (10).
5. Compute the required gain factors using equation (11).
6. Multiply each of the filtered sequences by its corresponding gain factor and add the scaled filtered sequences point by point to obtain the desired EEG signal.

CONCLUSION

In order to test the validity of digital analytical techniques for the EEG, simulated data, whose parameters are well defined, are generated. The principle of simulation is that the EEG can be represented by a rational transfer function, whose implementation is based on the passage of white noise through filters which occupy different frequency bands. The filter impulse response lengths are chosen to obtain the required characteristic, taking into account the effect of the window. Windowing the white noise is found to affect the δ–activity more than α and β activities. The spectral density function of the EEG is estimated by Welch method solely to obtain an approximate estimate of the parameters. Here the centre frequency parameters are more accurate than those for the bandwidth. Since the power parameters are calculated directly they are accurate. The SDF of individual activities are computed with the aim of testing whether the filtered outputs have the same characteristic as that of desired filter response. The filtered outputs have SDF which are close to their respective filter characteristics. Tests have shown that the generated data are accurate, in the sense that their characteristics resemble closely those of real EEG.

ACKNOWLEDGEMENT

The authors wish to acknowledge the interest and encouragement of Professor B.S. Sonde, Chairman, Electrical Communication Engineering Department, I.I.Sc., Bangalore. They would also like to thank Mr B.R.S. Reddy for his useful suggestions.

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