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Multi-period and multi-resource operating room scheduling under uncertainty: A case study

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ABSTRACT

Efficient planning and scheduling is essential for timely treatment of patients and improving the quality of operating room services and activities. In the present study, attempts are made to investigate a multi-period and multi-resource operating room integrated planning and scheduling problem under uncertainty. To this end, a mixed integer linear programming model has been developed for minimizing the tardiness in surgeries, overtime and idle time. Constraints related to human resources, equipment, as well as beds in pre-operative holding unit, recovery unit, ward and intensive care unit are taken into consideration. The durations of surgeries and recoveries are assumed uncertain, and a robust optimization approach has been used to manage the uncertainty. Due to the complexity of the model and the inability to solve large-scale problems, a metaheuristic method based on the genetic algorithm and a constructive heuristic approach have been proposed. After setting the parameters of the solution approaches using the Taguchi method, numerical experiments are performed based on various instances, and the results obtained from solving the mathematical model are compared to the results of the proposed metaheuristic and heuristic approaches. The results indicate that the proposed methods have a very good performance and the heuristic approach outperforms the genetic approach because the objective function of the proposed constructive heuristic is on average, about 19% better than the objective function of the genetic approach. A case study is also conducted in a hospital. The results obtained from the comparison of the proposed approaches with the hospital scheduling show that overtime and idle time are significantly improved in the proposed approaches.

1. Introduction

Hospitals are one of the fundamental elements of the healthcare industry. They consist of several units, such as a pharmacy, operating room, recovery unit, blood bank, laboratory, and radiology. Operating room is one of the most important and expensive units, and besides, it is a bottleneck resource (Landa, Aringhieri, Soriano, Tànfani, & Testi, 2016). Today, healthcare organizations are under pressure to offer surgical services at the lowest possible cost, and are facing challenges such as limited budgets, increasing waiting lists and aging populations at the same time (Molina-Pariente, Fernandez-Viagas, & Framinan, 2015). The demands for surgery are usually more than its supply, and this gives rise to long waiting times for patients and leads to reduced quality of services and dissatisfaction (Aringhieri, Landa, Soriano, Tànfani, & Testi, 2015). It is estimated that 60–70% of hospital admissions will require operating rooms (Van Essen, Hans, Hurink, & Oversberg, 2012). Operating rooms are very expensive. Surgical costs usually account for more than 40% of hospital costs (Denton, Viapiano, & Vogl, 2007). On the other hand, surgeries account for about 67% of hospital revenues. Of course, surgeries don't provide any revenue in many public hospitals in different countries (Saadouli, Jerbi, Dammak, Masmoudi, & Bouaziz, 2015).

Inattention to effective management and planning leads to delays, postponement of surgeries, withdrawal of patients, overtime, and eventually loss of revenue and decline in quality of treatment (Vancroonenburg, Smet, & Berghe, 2015). Timely treatment and increased efficiency and productivity in hospitals and operating rooms require proper management. Operating room planning and scheduling is one of the important aspects of operating room management which represents an application of optimization in the field of healthcare.

Operating room scheduling is associated with specific complexities due to the inherent uncertainties, various constraints and the presence of different beneficiaries. This issue has been one of the challenging research topics in the recent decades and years (Landa et al., 2016).

Different resources must be considered for proper scheduling and planning of operating theaters. Operating rooms, human resources

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(including surgeons, nurses, anesthesiologists, technicians, etc.), surgical equipment and tools, beds needed in preoperative holding unit (PHU), post-anesthesia care unit (PACU), intensive care unit (ICU) and ward are the resources that should be accessible and coordinated (Pham and Klinkert, 2008; Xiang, Yin, & Lim, 2015). According to Weinbroum, Ekstein, and Ezri (2003), the most important factors in waste of time in operating rooms are: lack of access to operating rooms 32%, lack of access to nurses 20%, lack of beds in the post-anesthetic care unit 10% and lack of access to specialist surgeon 4.7%. Therefore, about 70% of the waste of times in the operating rooms can be attributed to lack of coordination between resources and inaccessibility of them. Jonnalagadda, Walrond, Hariharan, Walrond, and Prasad (2005) found that the most important reasons for cancellation or delays in surgical procedures in developing countries include the lack of access to beds in the recovery unit (15%), problems in PHU (13%) and the lack of access to nurses (11%). These items demonstrate the importance of the downstream resources.

As mentioned before, operating room scheduling is associated with various uncertainties. The most important of which are uncertainty of the duration of surgery, postoperative length of stay, and arrival of emergency patients. Uncertainty of the surgery and post-operative length of stay has a direct effect on scheduling efficiency. The arrival of emergency patients (in case the resources for elective and emergency patients are not independent) causes disruptions in the plans. Uncertainty of the surgery duration can lead to overtime. Also, uncertainty in the length of stay in the recovery unit, intensive care unit, or ward may lead to cancellation of surgeries due to the shortage of beds (Neyshabouri and Berg, 2017).

Decisions made for management and planning of operating theaters are usually divided into three hierarchical levels: strategic, tactical, and operational. These items are fully described in the review articles in this area (Cardoen, Demeulemeester, & Beliën, 2010; Guerriero & Guido, 2011; May, Spangler, Strum, & Vargas, 2011) and we skip explaining them here. In the present paper, we focus on the operational level. Operating room scheduling at the operational level generally includes two main steps: the advance scheduling and allocation scheduling. The first step involves selection of patients from the waiting lists and assignment of specific date and operating room to them during the planning horizon. The second step provides a precise sequence of surgical procedures (determining start and finish times), along with allocation of resources to each patient. The former case is sometimes called operating room planning, and the latter is sometimes called operating room scheduling. In the literature, these two steps are presented as simultaneous (integrated) or hierarchical. Hierarchical scheduling reduces the quality of the acquired schedules due to the interdependence of the advance scheduling and allocation scheduling (Cardoen, Demeulemeester, & Beliën, 2009). Therefore, the interest in integrated planning and scheduling is increasing among researchers (Molina-Pariente et al., 2015; Roland, Di Martinelly, Riane, & Pochet, 2010; Van Huele and Vanhoucke, 2014). In this paper, an integrated approach has been used.

Many researchers distinguish between three management policies for the planning and scheduling of the operating rooms. Guerriero and Guido (2011) describe these three types of management policies including open, block, and modified block policies. In the block strategy, a set of time blocks are allocated to specific surgical specialties. In open strategy, time blocks do not belong to certain surgical groups and surgical procedures are planned based on the requests of surgeons. Open strategy performs significantly better than block strategy (Fei, Chu, Meskens, & Artiba, 2008; Van Huele and Vanhoucke, 2014). The modified block strategy is similar to the block strategy, except that some time blocks remain open in the schedule in order to provide more flexibility.

In this paper, the multi-period operating room planning and scheduling problem is studied with a comprehensive view of resources. Constraints on the number and availability of resources related to the surgical processes before and after surgery, including human resources (surgeons, anesthesiologists and nurses), beds of recovery unit, intensive care unit and ward as well as the required equipment and tools are taken into account. The uncertainty in the duration of surgery, recovery, and post-operative length of stay are also taken into consideration. Scheduling is done for elective patients under open scheduling strategy. A mixed integer linear programming model (MILP) has been developed and three criteria are considered for optimization: Idle time and overtime are considered as criteria based on hospital point of view, while the tardiness in surgery is a criterion based on patient point of view. Due to the complexity of the problem, a constructive heuristic (CH) algorithm and a hybrid genetic algorithm (GA) are proposed to solve medium-scale and large-scale problems. A case study is also conducted to investigate the efficiency of the mathematical model and the proposed algorithms in real conditions. The contributions of this paper are as follows:

- Developing a new mixed integer linear programming (MILP) model for multi-period integrated operating room planning and scheduling.
- Considering different units in the operating theater, including the PHU, the operating rooms, PACU, the intensive care unit and the ward.
- Providing a comprehensive view on the resources needed at the operating theater, including human resources, equipment and beds, and consideration of the limited number and availability of them for the first time.
- Considering uncertainty in the duration of surgery and recovery and using a robust optimization approach to manage uncertainty. Also, paying attention to the uncertainty in post-surgery length of stay.
- Proposing a new constructive heuristic algorithm for solving the problem.
- Implementing experimental analysis to compare the proposed approaches.

The remainder of this paper is classified as follows: Section 2 provides a review of relevant literature. Section 3 describes the problem and provides the mathematical model. Section 4 explains the proposed solution methods. Section 5 presents the computational results and analysis. A case study is presented in Section 6. Finally, Section 7 concludes and summarizes the study.

2. Literature review

Operating room planning and scheduling has been the subject of extensive studies. Review papers conducted by Cardoen et al. (2010), Guerriero and Guido (2011) and May et al. (2011) provide comprehensive literature review in this field. Accordingly, here we will review studies that are directly related to the subject of this article.

In the context of deterministic operating room scheduling, Jebali, Alouane, and Ladet (2006) introduced a two-stage approach consisting of sequencing and allocation for operating room scheduling. The authors proposed two mixed integer programming models. Constraints on resources such as surgeons, equipment and beds of recovery and intensive care units were taken into account. Roland et al. (2010) investigated the multi-period operating room planning and scheduling under the open scheduling strategy. They provided a mixed integer programming model. Constraints on access to renewable resources (including nurses, surgeons and some tools and equipment), as well as non-renewable resources (materials and some tools) were included in the problem, but the constraints of other units of the operating theater, such as the recovery unit, were not considered. The authors used genetic algorithm to solve the model. In a similar study, Silva, de Souza, Saldanha, and Burke (2015) studied the operating room scheduling with consideration of the similar factors, but on a daily basis. In their problem, a specialist human resource can simultaneously participate in more than one surgery. Vijayakumar, Parikh, Scott, Barnes, and Gallimore (2013) investigated the integrated and multi-period operating room planning and scheduling with consideration of limitations in access to nurses, surgeons and equipment. However, constraints related to other operating room units, including the recovery and intensive care units were not taken into account. They presented a mixed integer linear programming model with the objective of maximizing the number of surgical operations. They developed a heuristic method to solve the model. Meskens, Duvivier, and Hanset (2013) focused on daily scheduling of surgical operations with regard to access to human resources, their interdependence and preferences. They proposed a mathematical model with consideration of the mentioned items, as well as limitations on the recovery unit beds and the required equipment and tools. A similar study was also conducted by Wang, Meskens, and Duvivier (2015), in which mixed integer programming and constraint programming were employed. Considering the similarity between the daily operating room scheduling and the flexible job shop scheduling problem with limited multiple resources Xiang et al. (2015) developed a mathematical model with the aim of minimizing the duration of all surgeries. The flow of patients was taken into account in the pre-operative, intra-operative and post-operative stages. They used an ant colony optimization algorithm to solve the problem. Molina-Pariente et al. (2015) studied the multi-period integrated operating room planning and scheduling. They proposed an integer programming model with consideration of surgeons and assistant surgeons, and used an iterative constructive method to solve the problem. The authors assumed that the duration of the surgery depends on the experience of the surgical team, which can consist of a corresponding surgeon and an assistant surgeon. Vali Siar, Gholami, and Ramezanian (2017) studied the scheduling and re-scheduling problem of surgical operations with consideration of the constraints on access to surgeons and beds in the PHU and recovery units. They developed a mixed integer programming model and solved the problem with the rolling horizon approach.

In the aforementioned studies, the operating room scheduling problem is studied in the deterministic mode. As discussed in the introduction, uncertainty is inherent in the operating room planning and scheduling problem, and also exists in many factors such as the duration of surgery. In the following, we discuss the related articles that address the uncertainty in the duration of surgical processes. Researchers often use stochastic programming and robust optimization approaches to manage the uncertainty in the duration of surgical procedures in the operating room scheduling (Addis, Carello, Grosso, & Tànfani, 2016). Denton et al. (2007) considered the uncertainty in duration of surgeries and presented a two-stage stochastic programming model for the operating room scheduling problem. Taking into account the uncertainty in the duration of surgeries, Min and Yih (2010) used the two-stage stochastic programming approach for operating room scheduling in a multi-day planning horizon. They used the sample average approximation method to solve the problem. Lee and Yih (2014) modeled the operating room scheduling problem, with consideration of constraint on access to recovery beds, as a flexible job shop scheduling problem. They used fuzzy numbers for considering uncertainty in the duration of surgeries and solved the problem using a two-stage decision process and a genetic algorithm. Jebali and Diabat (2015) used two-stage stochastic programming for scheduling elective surgeries in a multi-day planning horizon with consideration of constraints of three hospital resources: operating rooms, beds in the intensive care unit (ICU) and beds in the ward. They considered the uncertainty of the surgery duration and the length of stay in the intensive care unit and the ward. The authors used the sample average approximation technique to solve the stochastic model. Two-stage stochastic programing has also been used in the paper of Heydari and Soudi (2016) in which the arrival of emergency patients was also taken into account.

Latorre-Núñez et al. (2016) studied the operating room planning and scheduling problem with consideration of constraints related to the required resources in the operating rooms and the recovery unit and the possibility of arriving emergency patients. They proposed an integer programming model a genetic-based metaheuristic method to provide a solution to the model. They also converted their model into a constraint programming model and ultimately compared the solutions obtained from the solution methods. Addis et al. (2016) provided an approach for the operating room scheduling and re-scheduling problem under uncertain surgery duration. They used the rolling horizon approach to deal with disruptions in scheduling and re-schedule the surgeries. The mathematical model presented by the authors was related to the operating room planning i.e. assigning days and operating rooms to patients. They presented the robust optimization of the problem using the robust optimization approach provided by Bertsimas and Sim (2004). Neyshabouri and Berg (2017) investigated the operating room planning problem with regard to uncertainties related to the duration of surgery and length of stay in the intensive care unit. They used the two-stage robust optimization approach based on the Bertsimas and Sim (2004) approach to consider the uncertainties. The authors used the column generation approach to solve the problem. The robust optimization technique is also used by Marques and Captivo (2017). In this paper, the authors considered the uncertainty in the surgery duration. Table 1 shows a summary of the articles related to the subject of the present study.

According to the literature and Table 1, many authors have not paid any attention to resources and have considered them unlimited, but this is not possible in the real world. In other articles some of the resources as well as their constraints and impacts have been addressed, but for a comprehensive and realistic planning and scheduling, it is necessary to take the limited resources into account. In recent years, researchers have focused on uncertainty in the operating room planning and scheduling. According to Table 1, more studies are needed in this area, especially because robust optimization techniques have been used in limited studies to manage uncertainty. In recent years, some authors have conducted studies in this field. In general, the cardinality-constrained robust optimization approach has rarely been addressed in the field of healthcare (Addis et al., 2016; Marques and Captivo, 2017). According to the literature, Addis et al. (2016), Marques and Captivo (2017) and Neyshabouri and Berg (2017) are among the few researchers that have used this approach for operating room planning and scheduling.

In order to fill the gaps, in this paper we have tried to consider all important resources in the operating theater. Uncertainty is considered in duration of surgery, recovery and length of stay in ICU/ward. Last but not least, we have developed a novel constructive heuristic algorithm to solve medium and large-scale problems.

In the present study we investigate the integrated multi-period operating room planning and scheduling problem with a comprehensive view of resources under uncertainty, in order to minimize tardiness of surgeries, as well as idle time and overtime.

3. Problem description

The surgical procedure for elective patients consists of three main stages. First, the patient is transported to PHU from the ward, and a bed is assigned to him/her. The patient stays in this unit until the corresponding nurse checks the patient's condition and documents and prepares him/her for surgery. After the necessary measures have been taken, the patient is transported to the assigned operating room (which has been prepared for surgery). Most hospitals have operating rooms with different sizes, features, and applications. Depending on the type of surgery, a suitable operating room with the necessary operational features should be assigned. Various resources are needed to perform surgery in the operating room. The required human resources include specialized surgeons, nurses (scrub nurse, circulator nurses and nurse anesthetists), anesthesiologists, and so on. In addition, sterilized tools and necessary equipment should be available. Obviously, the

Table 1
Summary of the related papers.

Research	Problem approach		Planning horizon		Resources								Uncertainty				Solution method	Criteria
	Scheduling	Planning	Daily	Multi-period	Surgeon	Nurse	Anesthesiologist	PHU beds	PACU beds	ICU beds	Ward beds	Equipment and tools	Surgery duration	Emergency arrivals	Recovery duration	LOS		
Jebali et al. (2006)	✓	✓		✓	✓				✓	✓		✓					1	6–7
Denton et al. (2007)	✓		✓										✓				3	1–7–8
Min and Yih (2010)	✓	✓		✓						✓			✓				1	6–7
Roland et al. (2010)	✓	✓		✓	✓	✓	✓					✓					1–2	3
Vijayakumar et al. (2013)	✓	✓		✓	✓	✓						✓					3	2
Meskens et al. (2013)	✓	✓	✓		✓	✓	✓		✓			✓					1	3–7–9
Lee and Yih (2014)	✓	✓	✓						✓				✓				2–3	3–8–9
Silva et al. (2015)	✓	✓	✓		✓							✓					1–3	4
Wang et al. (2015)	✓	✓	✓		✓	✓	✓		✓								1	3
Xiang et al. (2015)	✓	✓	✓		✓	✓	✓	✓	✓			✓					2–5	3
Molina-Pariente et al. (2015)	✓	✓		✓	✓								✓				1–3	2–4–9
Jebali and Diabat (2015)		✓		✓					✓	✓			✓			✓	1	7–8
Heydari and Soudi (2016)	✓	✓		✓					✓				✓	✓			1	3–7
Addis et al. (2016)		✓		✓									✓				1	5
Latorre-Núñez et al. (2016)	✓	✓	✓		✓	✓	✓		✓			✓	✓	✓			1–2	3
Neyshabouri and Berg (2017)		✓		✓	✓					✓			✓			✓	1	1–6–9
Vali Siar et al. (2017)	✓	✓		✓	✓			✓	✓								1	5–7–8
Marques and Captivo (2017)	✓	✓		✓	✓								✓				1	4
This research	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	1–2–3	5–7–8

Solution method: 1 – Mathematical programming (Goal programming, Column generation, Benders decomposition, MILP, ILP, MINLP, Dynamic programming, Branch and bound); 2 – Metaheuristics; 3 – Heuristics; 4 – Analytical and statistical methods; 5 – Simulation.// ***Criteria***: 1 – Patients' waiting time; 2 – Number of operated patients; 3 – Makespan/total completion time; 4 – Resource utilization; 5 – Postponing/canceling the surgeries; 6 – Financial objectives; 7 – Overtime; 8 – Idle time; 9 – Others.

composition of the surgical team and the necessary equipment and tools depend on the type and characteristics of the surgery. In the operating room, the patient is first anesthetized by an anesthesiologist and then the surgery is performed by the corresponding team. After the surgery, the patient is transported to the post-anesthesia care unit (recovery unit). A bed is assigned to him/her and is placed under the care of a nurse anesthetist. After that, in case the patient needs intensive care, he will be transported to the intensive care unit and a bed is assigned to him/her. The patient is taken care of and the necessary measures are taken for him/her and then transferred to the ward after passing the hospitalization period based on the specialist physician's opinion. Otherwise, he/she will be directly transported to the ward and will be discharged at the discretion of the corresponding physician when the hospitalization period is over. It should be noted that some patients may not need to be hospitalized and can be discharged shortly after surgery. Considering that several resources are needed for surgery and all surgeries are associated with many constraints, effective planning and scheduling is necessary for operating theaters. In this study, attempts are made to offer an efficient scheduling for reducing operating room overtime, idle time, and tardiness in surgeries, with consideration of the available resources and the existing constraints. Fig. 1 shows the necessary resources and different stages of a surgical procedure. The dashed arrows show that the patient may not enter the next stage.

In the present study, emergency surgeries are not taken into account. In reality, emergency surgical operations are often performed in different operating rooms, which are not used for elective patients. In addition, different surgeons are selected as corresponding emergency surgeons in different months and weeks (Augusto, Xie, & Perdomo, 2010). Due to the afore-mentioned reasons, emergency surgeries have not been taken into account in the present study. We have proposed a mixed integer linear programing model. The following conditions and assumptions are taken into account:

1. Time is discretized into 20-min intervals.
2. Each patient is operated at most once during the planning horizon.
3. A specialized surgeon and a suitable operating room are assigned to each surgery (if the surgery is planned to be done).
4. The team assigned to each surgery consists of a surgeon, an anesthesiologist, a circular nurse, a nurse anesthetist and a specified number of scrub nurses (surgeon assistants).
5. The list of surgical operations as well as other necessary information are input of the problem.
6. In the PHU and PACU a circular nurse and a nurse anesthetist will be assigned to each patient respectively. Given that in the intensive care unit and the ward, one nurse is usually assigned to several or all the available beds (depending on the unit size), the nurses assigned to these units are not included in the model.
7. Each surgery has a specific due date which gives priority to patients.

The indexes, parameters and decision variables of the mathematical model are defined as follows:

Sets

j Index for PHU ($j = 1$), operating room ($j = 2$) and recovery unit ($j = 3$)
k Index for intensive care unit ($k = 1$) and ward ($k = 2$)
p Index for patients requiring surgery; p: 1, 2, …, P
s Index for surgeons; s: 1, 2, …, S
o Index for operating rooms; o: 1, 2, …, O
d Index for days of the planning horizon; d: 1, 2, …, D
d_{ex} Index for extended set of days of the planning horizon (For considering days exceeding the planning horizon): d_{ex}: 1, 2, …, D_{ex}
t Index for time slots during day; t: 1, 2, …, T
e Index for equipment; e: 1, 2, …, E

Parameters

RT The last time slot in the regular opening hours (which is The last time slot that is not considered as overtime)
OT The last time slot in the opening hours (RT plus maximum allowed overtime hours)
du_{pj} Duration of stage j for patient p
dt_p Latest day to perform surgery p (similar to due date in production scheduling)
OS_{std} A binary parameter; 1, if surgeon s is not available at time t on day d; 0, otherwise.
Nu_{td}^{c} Total number of circular nurses available at time t on day d
Nu_{td}^{at} Total number of nurse anesthetists available at time t on day d
Nu_{td}^{sc} Total number of scrub nurses available at time t on day d
Nu_{td}^{as} Total number of anesthesiologist available at time t on day d
Eq_{etd} Total number of available equipment type e at time t on day d
α_{ep} Number of equipment type e required for the patient p
λ_p The number of scrub nurse required for patient p
B_{pos} A binary parameter; 1, if surgery of patient p can be performed by surgeon s in the operating room o; 0, otherwise
$H_s^{\max}$ The maximum number of time slots which surgeon s can perform surgery in the entire planning horizon

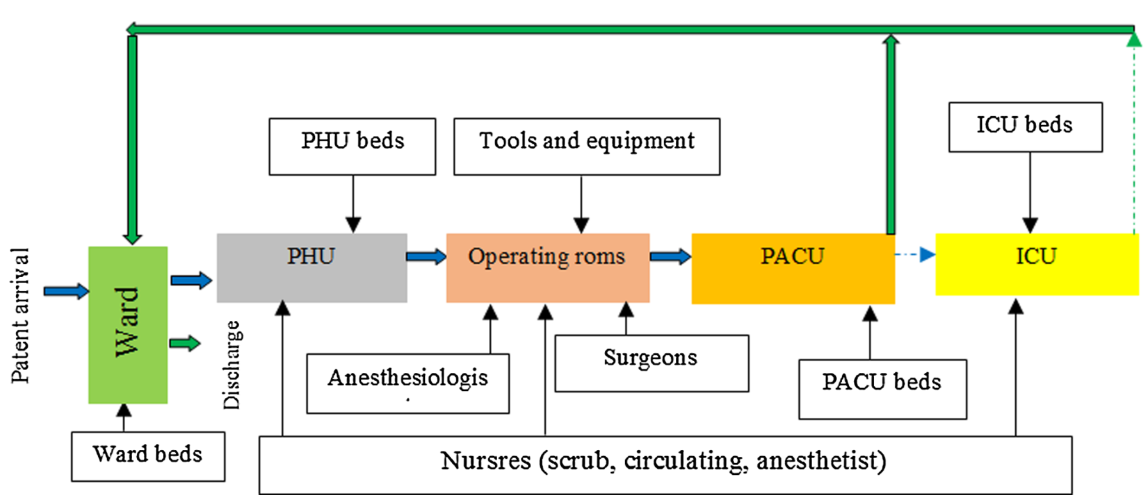

Fig. 1. Stages and resources in performing surgeries.

Los_{pk} Length of stay of patient p after operation in ICU ($k = 1$) or ward ($k = 2$)
wb_p Number of days that patient p stays in the ward before surgery
Bed_j Total number of beds available in PHU ($j = 1$) and the recovery unit ($j = 3$)
Bed^{ward} Total number of beds available in ward
Bed^{ICU} Total number of beds available in ICU
θ_{dex}^{ICU} Number of patients staying in the ICU on day d_{ex} from the previous planning horizon
θ_{dex}^{ward} Number of patients staying in the ward on day d_{ex} from the previous planning horizon
w_T Weighted factor for the minimization of tardiness of surgeries
w_O Weighted factor for the minimization of overtime
w_I Weighted factor for the minimization of idle time

Decision variables

x_{postd} A binary variable; 1, if patient p is scheduled in OR o with surgeon s at time t on day d; 0, otherwise
y_{pod} A binary variable; 1, if patient p is scheduled in OR o on day d; 0, otherwise
δ_{ptdj} A binary variable; 1, if phase j of surgical operation of patient p is scheduled at time t on day d; 0, otherwise
zs_{ps} A binary variable; 1, if patient p is assigned to surgeon s; 0, otherwise
ts_{pj} Start time of phase j for patient p
tc_{pj} Finish time of phase j for patient p
$v_{pd_{ex}}$ A binary variable; 1, if patient p stays in ward on day d_{ex} (before surgery).
$\gamma_{pd_{ex}k}$ A binary variable; 1, If patient p stays in ICU ($k = 1$) or ward ($k = 2$) on dayd_{ex}
ovt_{od} The overtime of operating room o on day d
idt_{od} The idle time of operating room o on day d

3.1. Deterministic mathematical model

In the mathematical model of this study, three criteria are minimized in the objective function. These criteria include: idle time, overtime and tardiness of surgeries. The precise modeling of the operating room planning and scheduling problem may vary considerably from one hospital and surgery department to another (Riise, Mannino, & Burke, 2016). In the present study, attempts are made to offer a comprehensive and generalized modeling with maximum applications in different circumstances.

$$Min = \frac{w_T}{D*P}\left(\sum_{\substack{p\\ dt_p\leqslant D}}\sum_{o}\sum_{\substack{d\\ d>dt_p}}(d-dt_p)y_{pod} + \sum_{\substack{p\\ dt_p<D}}(D-dt_p+1)\left(1-\sum_{o}\sum_{d}y_{pod}\right)\right) + \frac{w_O}{O*D*(OT-RT)}\left(\sum_{o}\sum_{d}ovt_{od}\right) + \frac{w_I}{O*D*RT}\left(\sum_{o}\sum_{d}idt_{od}\right) \quad (1)$$

Subject to:

$$\sum_{o}\sum_{s}\sum_{d}x_{postd}\leqslant 1 \quad \forall\, p, t \quad (2)$$

$$\sum_{p}\sum_{s}x_{postd}\leqslant 1 \quad \forall\, o, t, d \quad (3)$$

$$\sum_{p}\sum_{o}x_{postd}\leqslant 1 \quad \forall\, s, t, d \quad (4)$$

$$\sum_{p}\sum_{o}\sum_{s}\sum_{t>OT}\sum_{d}x_{postd}=0 \quad (5)$$

$$\sum_{o}\sum_{d>wb_p}y_{pod}\leqslant 1 \quad \forall\, p \quad (6)$$

$$\sum_{s}\sum_{t}x_{postd}\leqslant T*y_{pod} \quad \forall\, p, o, d \quad (7)$$

$$\sum_{o}\sum_{d}y_{pod}=\sum_{s}zs_{ps} \quad \forall\, p \quad (8)$$

$$\sum_{o}\sum_{s}\sum_{t}\sum_{d}x_{postd}\geqslant du_{p2}*\sum_{s}zs_{ps} \quad \forall\, p \quad (9)$$

$$\sum_{o}x_{postd}\leqslant(1-OS_{std})*zs_{ps} \quad \forall\, p, s, t, d \quad (10)$$

$$\sum_{t}\sum_{d}x_{postd}\leqslant T*B_{pos} \quad \forall\, o, p, s \quad (11)$$

$$\delta_{ptd2}=\sum_{o}\sum_{s}x_{postd} \quad \forall\, p, t, d \quad (12)$$

$$\delta_{ptdj}\leqslant\sum_{o}y_{pod} \quad \forall\, p, t, d, j \quad (13)$$

$$\sum_{t}\sum_{d}\delta_{ptdj}\geqslant du_{pj}*\sum_{o}\sum_{d}y_{pod} \quad \forall\, p, j \quad (14)$$

$$ts_{pj}=tc_{p(j-1)} \quad \forall\, p, j\colon j\neq 1 \quad (15)$$

$$ts_{pj}\leqslant t*\sum_{d}\delta_{ptdj}+T*(1-\sum_{d}\delta_{ptdj}) \ \forall\, p, t, j \quad (16)$$

$$tc_{pj}\geqslant(t+1)*\sum_{d}\delta_{ptdj} \quad \forall\, p, t, j \quad (17)$$

$$\sum_{t}\sum_{d}\delta_{ptdj}=tc_{pj}-ts_{pj} \quad \forall\, p, j\colon j\neq 2 \quad (18)$$

$$\sum_{p}\delta_{ptdj}\leqslant Bed_j \quad \forall\, t, d, j \quad (19)$$

$$\sum_{d_{ex}=d-wab_p}^{d-1}v_{pd_{ex}}=wb_p*\sum_{o}y_{pod} \quad \forall\, p, d \quad (20)$$

$$\sum_{d'_{ex}=d}^{d+Los_{p1}-1}\gamma_{pd'_{ex}1}=Los_{p1}*\sum_{o}y_{pod} \quad \forall\, p, d \quad (21)$$

$$\sum_{d'_{ex}=d+Los_{p1}}^{d+Los_{p1}+Los_{p2}-1}\gamma_{pd'_{ex}2}=Los_{p2}*\sum_{o}y_{pod} \quad \forall\, p, d \quad (22)$$

$$\sum_{p}\gamma_{pd_{ex}1}+\theta_{d_{ex}}^{ICU}\leqslant Bed^{ICU} \quad \forall\, d_{ex} \quad (23)$$

$$\sum_{p}\gamma_{pd_{ex}2}+\theta_{d_{ex}}^{ward}\leqslant Bed^{Ward} \quad \forall\, d_{ex} \quad (24)$$

$$\sum_{p}\sum_{j<3}\delta_{ptdj}\leqslant Nu_{td}^{c} \quad \forall\, t, d \quad (25)$$

$$\sum_{p}\lambda_p*\delta_{ptd2}\leqslant Nu_{td}^{sc} \quad \forall\, t, d \quad (26)$$

$$\sum_{p}\sum_{j>1}\delta_{ptdj}\leqslant Nu_{td}^{at} \quad \forall\, t, d \quad (27)$$

$$\sum_{p}\delta_{ptd2}\leqslant Nu_{td}^{as} \quad \forall\, t, d \quad (28)$$

$$\sum_{p} \alpha_{ep} * \delta_{ptd2} \leqslant Eq_{etd} \quad \forall\ t, d, e \tag{29}$$

$$\sum_{p}\sum_{o}\sum_{t}\sum_{d} x_{postd} \leqslant H_s^{\max} \quad \forall\ s \tag{30}$$

$$idt_{od} = RT - \sum_{p}\sum_{s}\sum_{\substack{t \\ t \leqslant RT}} x_{postd} \quad \forall\ o, d \tag{31}$$

$$ovt_{od} = \sum_{p}\sum_{s}\sum_{\substack{t \\ t > RT}} x_{postd} \quad \forall\ o, d \tag{32}$$

$$\begin{aligned} & x_{postd} \in \{0, 1\} \quad \forall\ p, o, s, t, d\text{: } d > dt_p \\ & \delta_{pdtj} \in \{0, 1\} \quad \forall\ p, t, d, j \\ & \gamma_{pd_{ex}k} \in \{0, 1\} \quad \forall\ p, d, k \\ & ts_{pj}, tc_{pj} \in \{0, I^{+}\} \quad \forall\ p, j \\ & idt_{od}, ovt_{od} \in \{0, I^{+}\} \quad \forall\ o, d \end{aligned} \tag{33}$$

The objective function (1) consists of three parts. The first part minimizes the tardiness of surgeries. The second and third parts are related to minimization of overtime and idle time of the operating rooms respectively. Constraint (2) states that at each time slot, a maximum of one operating room and one surgeon can be assigned to each patient (surgery) during the planning horizon. Constraint (3) ensures that a maximum of one surgery can be performed in an operating room on each day. Constraint (4) ensures that each surgeon can perform one surgery at a given time slot per day. Constraint (5) states that no surgery can be performed after the last time slot that ORs are permitted to be active. Constraint (6) ensures that each patient can be assigned to one operating room and is scheduled at most once during the planning horizon. Also, this constraint allows a surgery of a patient only if he/she can stay in the ward wb_p days before surgery. Constraints (7) and (8) contribute to determination of variables y_{pod} and zs_{ps} that are required in the objective function and the other constraints. Constraint (9) ensures that sufficient number of time slots is assigned to each surgery according to its duration. Constraint (10) ensures that the surgeon assigned to a surgery is available at the scheduled time and day. Constraint (11) ensures that the surgeon has the specialty for performing the surgery, and the assigned operating room is suitable for the surgery. Constraints (12) and (13) help determine the value of variable δ_{ptdj} that is required in other constraints and necessary for determining the correct schedule and sequence between PHU, operating room, and PACU. Constraint (13) states that if the surgery of patient $\bar{p}$ is scheduled on day $\bar{d}$ ($y_{\bar{p}o\bar{d}} = 1$) then $\delta_{\bar{p}\bar{t}\bar{d}j}$ can be 0 or 1. It means that $\delta_{\bar{p}\bar{t}\bar{d}j}$ is surely 0 for the days other than $\bar{d}$. Constraint (14) states that the total number of assigned time slots at each stage is sufficient with regard to the associated duration. Following the explanations presented for constraint (13), if the surgery of patient $\bar{p}$ is scheduled ($y_{\bar{p}od} = 1$), $\delta_{\bar{p}tdj}$ must be equal to 1 for a day d, then based on constraint (13) d should be equal to $\bar{d}$, because $\delta_{\bar{p}\bar{t}\bar{d}j}$ is zero for the other days. So, staying in the PHU, performing surgery, and staying in the PACU are sequenced and performed on one single day that is obviously logical. Constraint (15) is related to the sequencing of afore-mentioned three stages, and states that the start time of each stage is equal to finish time of the previous stage; that is, there is no waiting time between two consecutive stages. This constraint guarantees that the stage related to the surgery ($j = 2$) starts after finishing the stage related to PHU ($j = 1$), and also recovery ($j = 3$) starts after completion of surgery. Constraints (16) and (17) determine the start and finish times for each stage respectively. Constraint (18) warrants that the difference between the start and finish time at each stage is equal to the assigned time slots. Constraint (19) ensures that, at each time slot on a given day, the total number of patients staying in the PHU or PACU should not exceed the number of beds available in these units. Constraint (20) determines the days that patient stay in the ward before surgery. Constraints (21) and (22) determine the days in which the patients stay in ward or ICU after the surgery. Considering the capacity of ICU and ward and the number of patients staying in these units from the previous planning horizons, constraints (23) and (24) limit the number of patients who can stay in the ICU and ward on each day. Constraints (25)–(29) are related to res the limitations of on the number of nurses, anesthesiologists and equipment. Constraint (30) limits the maximum number of time slots which a surgeon can perform surgery during the planning horizon. Constraints (31) and (32), compute the idle time and overtime time of each OR on each day. Finally, constraint (33) delineates the type and sign of decision variables. I^{+} denotes the set of positive integer numbers.

3.2. The model under uncertainty

In the present study, the durations of surgery and recovery are considered uncertain in order to make the conditions as realistic as possible. We have applied a robust optimization approach. The main reason for using this approach is that it is very difficult to obtain and construct a probability distribution in most of the times. On the other hand, the probability distribution may not be appropriate for building scenarios in the short term planning. In other words, the probability distribution of surgical and recovery durations for a small group of patients in the next week may be completely different from the probability distribution obtained using data from several years. The robust optimization method provides tools for obtaining solutions that are protected against uncertainty. This feature is very important in the field of healthcare, including operating room planning, because planning without regard to uncertainty would have serious impacts on the health of patients. In two-stage stochastic programming, evaluation of the second stage is required for each scenario, and if the number of scenarios is high, solving issues may become problematic in terms of tractable and intractable solution methods. The risk-averse nature of robust optimization method makes it an appropriate framework because it inherently caters towards patient safety and focuses on high-quality care and treatment (Neyshabouri and Berg, 2017).

In order to deal with uncertainty, in this section, the robust model is presented based on the robust optimization approach proposed by Bertsimas and Sim (2004). In the model described in Section 3, constraints (9) and (14) include uncertain parameters and can be rewritten as follows:

$$\sum_{o}\sum_{s}\sum_{t}\sum_{d} x_{postd} \geqslant d\tilde{u}_{p2} * \sum_{s} zs_{ps} \quad \forall\ p \tag{34}$$

$$\sum_{t}\sum_{d} \delta_{ptdj} \geqslant d\tilde{u}_{pj} * \sum_{o}\sum_{d} y_{pod} \quad \forall\ p, j \tag{35}$$

Now, the formulation provided by Bertsimas and sim (2004), can be used to extract the robust formulation of these two constraints. Here, du_{pj} and $d\hat{u}_{pj}$ are nominal values and the maximum deviation from the nominal value, respectively. Γ_{pj} which is the budget of uncertainty is denoted by Γ_j (in other words, it is the same for all patients) and according to the constraints we have $\Gamma_j \in [0, 1]$. The constraints (36)–(41) are robust counterparts of constraint (34):

$$du_{p2} * \sum_{s} zs_{ps} + \Gamma_2 z_p + q_p \leqslant \sum_{o}\sum_{s}\sum_{t}\sum_{d} x_{postd} \quad \forall\ p \tag{36}$$

$$z_p + q_p \geqslant d\hat{u}_{p2} * \sum_{s} u_{ps} \quad \forall\ p \tag{37}$$

$$-\sum_{s} u_{ps} \leqslant \sum_{s} zs_{ps} \leqslant \sum_{s} u_{ps} \quad \forall\ p \tag{38}$$

$$z_p \geqslant 0 \quad \forall\ p \tag{39}$$

$$q_p \geqslant 0 \quad \forall\ p \tag{40}$$

$$u_{ps} \geqslant 0 \quad \forall\ p, s \tag{41}$$

In the above constraints, z_p and q_p are the variables required to build the robust counterpart. The robust counterpart of constraint (35) is also written via constraints (42)–(47) according to the formulation provided by Bertsimas and Sim (2004).

$$du_{pj} * \sum_{o} \sum_{d} y_{pod} + \Gamma_j z'_{pj} + q'_{pj} \leqslant \sum_{t} \sum_{d} \delta_{ptdj} \quad \forall\, p, j \tag{42}$$

$$z'_{pj} + q'_{pj} \geqslant d\hat{u}_{pj} * \sum_{o} \sum_{d} u'_{pod} \quad \forall\, p, j \tag{43}$$

$$-\sum_{o} \sum_{d} u'_{pod} \leqslant \sum_{o} \sum_{d} y_{pod} \leqslant \sum_{o} \sum_{d} u'_{pod} \quad \forall\, p \tag{44}$$

$$z'_{pj} \geqslant 0 \quad \forall\, p, j \tag{45}$$

$$q'_{pj} \geqslant 0 \quad \forall\, p, j \tag{46}$$

$$u'_{pod} \geqslant 0 \quad \forall\, p, o, d \tag{47}$$

In the above constraints, z_p, q_p and u'_{pod} are robust variables. Of course, the constraints (38), (41), (44) and (47) can be removed because the variables are binary and will never be negative.

4. Solution methods

In this section, the complexity of the problem is addressed, and then the proposed solution methods are described. In this paper, a metaheuristic based on genetic algorithm, and also a constructive heuristic algorithm are developed.

4.1. The problem complexity

The proposed mixed integer programming model discussed in Section 3 is based on the model proposed by Vijayakumar et al. (2013). They studied the scheduling of elective patients considering operating rooms, surgeons and nurses. In detail, the assignment of patients to operating rooms, days and surgeons and determining the start and finish times of surgeries are the decisions made in their model. If we omit our decision variables and constraints related to considering PHU, PACU, ICU and ward and objectives associated to minimization of idle time and overtime, the problem can approximately be reduced to the one proposed by Vijayakumar et al. (2013) and all the mentioned decisions are made in our model too. Vijayakumar et al. (2013) have expressed that their proposed model is NP-hard via comparing their problem with the dual bin packing problem. Therefore, the problem presented in this paper which has more decision variables and constraints and has considered other stages of operating theatre is NP-hard too. In addition, operating room scheduling problem considering different stages can be seen as a flexible flow-shop scheduling problem (Latorre-Núñez et al., 2016), which is actually an NP-hard problem (Gupta, 1988). Our preliminary tests suggest that the MILP model was able to solve the problems up to 20 patients, 5 operating rooms, 5 surgeons and a planning horizon of 6 days. The computational tests are performed on a computer with 8 GB of RAM and an Intel (R) Core (TM) i72630QM, 2 GHz CPU, running on Windows 8.1 (64-bit), in a suitable CPU time. The mentioned problem instances are not acceptable in comparison with real world problems, on the other hand, it is necessary to create efficient operating room schedules in reasonable CPU times. Therefore, heuristic and metaheuristic solution methods are needed to solve large-scale problems. In the following, we introduce the proposed solution methods.

4.2. Genetic algorithm

The genetic algorithm was derived from research on cellular automata in the 1970s and was first introduced by Holland (1975). As its name implies, this algorithm is inspired by biological evolution. Genetic algorithms are a popular class of evolutionary algorithms and utilize some rules derived from evolutionary biology including crossover, mutation and selection (Roland et al., 2010). GA is a constructive and population-based algorithm. In this algorithm, each candidate solution is shown by a chromosome. In the other words, each chromosome is a representation of a candidate solution. The solution representation differs from a problem to another. The population of GA consists of chromosomes and in an iterative process the population evolves towards a higher quality population. The initial step of the algorithm includes generating a population randomly (*POP*) containing a specific number (n_{pop}) of individuals (solutions). In each iteration, the quality of the individuals is evaluated via fitness function. In order to evolve the population, in an iterative process, the algorithm implements crossover and mutation operators on a certain percentage of individuals to obtain higher-quality individuals. The function of crossover operator is to inherit some characteristics of the two parents to generate the offsprings or children. Also, a certain number of individuals are affected by mutation. This operator acts on a single individual in a way that one or more genes of the chromosome of individuals are mutated (Talbi, 2009). In the last step of each iteration the algorithm select n_{pop} individuals which have better quality based on fitness function values and then the next iteration starts. The algorithm is repeated until a stopping criterion is met which can be a CPU time limit or reaching to a specific number of iterations. The steps involved in the implementation of the proposed genetic algorithm are discussed below.

4.2.1. Solution representation

Each solution (population member) is represented by three uncorrelated chromosomes as $I = (\delta, \mu, \sigma)$, where δ is the list of surgeries along with the day assigned to them. σ and μ indicate the operating rooms and surgeons assigned to the surgeries respectively. A similar approach is used in Roland et al. (2010). The solution is represented as follows:

$$I = \begin{pmatrix} \delta \\ \sigma \\ \mu \end{pmatrix} = \begin{pmatrix} (p_1, d_1) & \dots & (p_P, d_P) \\ o_1 & \dots & o_P \\ s_1 & \dots & s_P \end{pmatrix} \tag{48}$$

The couple (p_i, d_i) of δ means operation of patient p_i is scheduled on day d_i. The operating room o_i and surgeon s_i are also determined by σ *and* μ respectively. The surgical operations on a given day will be arranged according to the order of surgeries determined in δ. For example, assume that 6 patients exist in the waiting list. The planning horizon has 3 days, the number of operating rooms is 2, and the number of surgeons is 3. A generated solution can be represented as Fig. 2. For example, this figure shows that patient 4 should be operated on day 2 in the operating room 2 by surgeon 1.

4.2.2. Generating initial population

The initial population in the genetic algorithm is usually generated randomly, but sometimes, due to the existence of numerous constraints, to avoid generation of too many non-feasible solutions and increase the convergence rate of the solutions, heuristic algorithms are used. The heuristic algorithm used in this study acts as follows: at first, surgical operations (a total of P surgeries) are randomly arranged. Then,

δ	$p_1=1$	$d_1=2$	$p_2=6$	$d_2=3$	$p_3=4$	$d_3=2$	$p_4=2$	$d_4=3$	$p_5=5$	$d_5=1$	$p_6=3$	$d_6=1$
σ	$o_1=1$		$o_2=2$		$o_3=2$		$o_4=1$		$o_5=1$		$o_6=2$	
μ	$s_1=1$		$s_2=2$		$s_3=1$		$s_4=3$		$s_5=2$		$s_6=3$	

Fig. 2. Solution representation of the proposed genetic algorithm.

Generating random solutions

Input: list of patients who require surgery, the number of days of the planning horizon, matrix B_{pos}, matrix OS_{std}, data of patients' staying in ward before surgery and staying in the ICU/ward after surgery, the duration of surgeries, matrix of consumed time $Cons\_time_{od}$

output: initial population with size of $npop$ (based on the structure of solution)

```
Begin
    Step 0.
    sort patients randomly.
    Step 1.
    for i=1 to P do
        select a surgeon randomly using matrix B_pos and assign to patent i (s_i).
        select an operating room randomly using matrix B_pos and assign to patent i (o_i).
        select a day randomly using matrix OS_std and assign to patent i (d_i). (For Patients who need
        to stay in ward before surgery d_i = wab_i + 1.
        while Cons_time_{o_i d_i} + du_i2 > OT do
            repeat above steps.
        end
        update matrix of consumed time.
    end
end
report generated random solution.
```

Fig. 3. Pseudo-code of generating initial population.

according to the specialties of surgeons specified in the matrix B_{pos}, surgeons are randomly assigned to the surgeries. The day of each surgery is randomly selected among the days that the assigned surgeon is available in the hospital considering matrix OS_{std}. In addition, the length of stay in the hospital before surgery should be taken into consideration. For each surgery, matrix B_{pos} is used to randomly assign an operating room among all of the suitable operating rooms, provided that the total amount of time used in that operating room on the assigned day ($Cons\_time_{o_i d_i}$), plus the duration of the intended surgery (du_{i2}), do not exceed the total time allowed of the operating room (OT). Accordingly, the initial solution generation process is supposed to provide feasible solutions as much as possible. Using this process, the generated population size should be as large as n_{pop}. The pseudocode of this algorithm is presented in Fig. 3.

4.2.3. Decoding and feasible maker procedures

As indicated in the solution representation structure, the time allocated to surgical operations is not specified in the structure. In this research, a heuristic algorithm is used to allocate the solutions to the time axis. This algorithm is, in fact, a feasible maker procedure that tests different constraints to avoid generation of infeasible solutions as much as possible. Some patients may not be operated due to unavailability of resources on the assigned day and be canceled. If the necessary conditions are provided for each patient, depending on the assigned day and operating room, the vacant time slots (starting from the first vacant time slots) will be assigned to the surgery in question. The pseudocode of the described heuristic algorithm is presented in Fig. 4.

As can be observed in Fig. 4, in an iterative process, all (p) patients are considered and each surgery will be scheduled considering the input data from the step of generating random solutions, provided that it is possible due to different constraints. If a surgery is to be scheduled, it will be registered in the matrix $Schedule_{odt}$ which is a three dimensional (lengths of dimensions are O, D and OT respectively) matrix and all of its elements are initially zero. Then in each iteration, the relevant elements will be assigned to a surgery that should be scheduled based on its assigned day, operating rooms and time slots. The purpose of updating the status of the resources in Fig. 4 is to change the status of the surgeon to unavailable while he/she performs a surgery, reduce the number of available anesthesiologists, nurses and equipment based on the amount used, and finally update the number of beds available in PHU, recovery, intensive care unit and ward according to the time or days assigned to the patients. Another point is that the time index (T_index_{od}) indicates the time slot that a surgery can start from. Initially, the value of this index is 1 for all days and all operating rooms and is updated during the implementation of the heuristic algorithm.

4.2.4. Evaluation

The value of the solutions generated at each stage should be investigated by a fitness function. The fitness function used in this research is the same as the objective function of the problem which is as follows:

$$Min = \frac{w_T}{D*P}\left(\sum_{\substack{p\\ dt_p \leqslant D}} \sum_{o} \sum_{\substack{d\\ d > dt_p}} (d-dt_p)y_{pod} + \sum_{\substack{p\\ dt_p < D}} (D-dt_p + 1)\left(1-\sum_{o}\sum_{d} y_{pod}\right)\right) + \frac{w_O}{O*D*(OT-RT)}\left(\sum_{o}\sum_{d} ovt_{od}\right) + \frac{w_I}{O*D*RT}\left(\sum_{o}\sum_{d} idt_{od}\right) \quad (49)$$

4.2.5. Parent selection

In this paper, the roulette wheel selection is used for parent selection. In this method, the following selection function is used:

$$P = e^{(-\beta*Costs/Worst\_Cost)} \quad (50)$$

Decoding (feasible maker procedure)

Input: generated solutions by *Generating initial population* procedure/solutions affected by crossover or mutation, data related to duration of different stages, data related to accessibility and number of recourses, data related to number of required resources for each patient and matrix $Schedule_{odt}$, time index vector T_index_{od}

output: initial and feasible population with size of $npop$

```
for i=1 to P do
    if T_index_{o_i d_i} + du_{i2} ≤ OT
        if surgery i can be scheduled in assigned operating room o_i on determined day d_i and operated
        by assigned surgeon s_i in period t = T_index_{d_i} to t = T_index_{d_i} + du_{i2} − 1, satisfying all
        constraints then
            Assign possible time slots to surgery i and update Schedule_{odt}.
            Register the completion time of surgery i (tc_i).
            update the status of all resources.
            update time index vector: T_index_{o_i d_i} = tc_i + 1.
        end
    end
end
report initial feasible solution.
```

Fig. 4. Pseudo-code of decoding procedure (feasible maker procedure).

where the parameter β is adjustable and is often selected in such a way that the probability of the first half of the population members is about 0.8 (at first the population is ranked according to the fitness function, and then the above function is used. therefore the first half of the population is actually the first half of the ranked population).

4.2.6. Operator

In this section, two operators (crossover and mutation operators) used in the proposed genetic algorithm are described.

4.2.6.1. Crossover. The parents selected by the roulette wheel selection procedure are used to generate new members by the Crossover operator. This operator is applied to a certain number of population members (adjustable), and a new member is generated in each stage according to the mechanism described below.

In the present study, partially mapped (PMX) crossover is used (Buckles, Petry, & Kuester, 1990). This operator, first randomly selects two numbers to specify the crossing locations. The alleles of the first parent that fall between the two crossover sites are copied into the same positions of the offspring. The positions of other alleles of the offspring are determined by the second parent according to a two-step process. First, the alleles of the second parent which are not within the crossing sites are copied to the corresponding positions of the offspring. Next, for each allele of the second parent within the crossing sites, the position of allele from first parent that has displaced it is determined in the second parent and the allele is placed in the same position in the offspring.

In our paper, the described operator is applied to patients (Part 1 of the first chromosome in each solution) and the genes of other chromosomes are also determined simultaneously. In the other words, the day, the surgeon and the operating room assigned to each patient are copied from the parent who has entered the allele related to patient to the offspring chromosome. Fig. 5 shows how the crossover works. Considering this figure, surgeries 7, 9, 2 and 6 (number of patients and their corresponding information including assigned day, operating room and surgeon) are directly copied from the first parent into the same positions of the offspring. Surgeries 8, 1 and 5 are copied from the second parent into the same positions of the offspring. Surgery 3 in the second parent is displaced by surgery 7 from first parent. As can be seen, the position of surgery 7 is determined in the second parent (denoted by "*") and surgery 3 is placed in the specified position. A same process occurs for surgery 4.

4.2.6.2. Mutation. Just Like the crossover operator, this operator is applied to a certain number of population members. First, for the patients $i = 1, 2, ..., P$ the surgeries p_i and p_{i+1} are replaced with each other with probability P_{mut} so that the day, the operating room and the surgeon assigned to each operation remain intact. Then, the patients $i = 1, 2, ..., P$, are assigned to new days, operating rooms, and the surgeons with the same probability (considering the points mentioned for generating the initial population).

Fig. 6 shows the general scheme of the genetic algorithm in form of a pseudocode. At first, the GA main parameters are determined. Then, in the first step (Step 1), an initial population is generated randomly. After that, the iterative process (Step 2) begins; in each iteration crossover and mutation operators are applied (Steps 2.1 and 2.2) and in the last step (Step 2.3), a sufficient number of solutions (*npop*) is selected based on fitness function values. Then, the next iteration starts. The termination criterion of the proposed algorithm is to reach a repetition threshold.

4.3. Constructive heuristic algorithm

The constructive heuristic algorithm consists of two main phases. In the first phase, an initial schedule with highest possible quality is generated. In the second step, the schedule obtained from the first phase is improved. This algorithm is repeated for a certain number of generations and the best solution is reported as the final solution. Just like the genetic algorithm, the fitness function used in this algorithm is the objective function of the problem (Eq. (51)).

4.3.1. Phase 1 of the heuristic Algorithm: generation of initial schedule

In the first phase, patients are randomly ordered (*WL*). Then, in the second step, the algorithm goes to the first patient in the randomly ordered list, and assigns each patient to a specific day with consideration of all possible modes, i.e. all modes in which the patient can be operated in different operating rooms by different surgeons in the

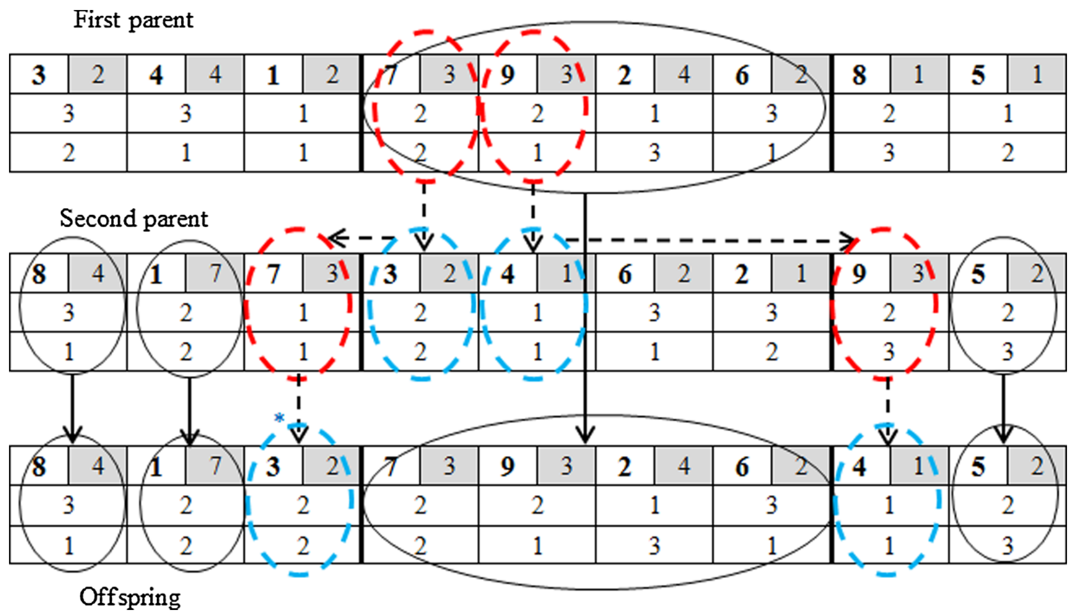

Fig. 5. Implementing crossover via PMX approach.

earliest possible time. All of the found schedules should be saved in a structure that is denoted by *D_SCH*. Then, the best possible schedule is selected for each day, depending on the value of fitness function. Finally, after obtaining a schedule for all potential days, the best schedule is selected (*Schedule*) and the list of operated patients (*ope*) and the status of resources is updated according to this schedule. The pseudocode of this phase is presented in Fig. 7.

4.3.2. Phase 2 of the heuristic algorithm: improving the initial schedule

The role of the second stage is to improve the initial schedule created in the first phase. This phase consists of three main steps. In the first step, the algorithm begins with patient number 1, and eliminates the patient from the schedule if it is partly or totally scheduled in the overtime. The new schedule is then evaluated and replaced with the initial schedule if it has a higher quality and the patient is removed from the list of scheduled patients. In the second step, patients who are not scheduled are identified and are then scheduled step-by-step if possible in order to decrease idle time and also to avoid tardiness. In case the fitness function is improved, the new schedule replaces the previous one. In the third step, a specific number of surgeries are randomly selected and removed from the schedule and put in the list of removed surgeries (*re*). Then they are rescheduled at other possible times and days. Finally, the schedule is evaluated and replaced with the initial schedule if it has higher quality. The pseudocode of the improving phase (second phase) is shown in Fig. 8.

The pseudocode of the proposed CH algorithm is shown in Fig. 9.

4.4. Setting the parameters of the proposed algorithms

In this section, we draw on the Taguchi method to tune the parameters of the two proposed algorithms using the Minitab 16 software. The proposed genetic algorithm includes 5 parameters that need to be set. Three levels are considered for each parameter. Table 2 shows these parameters and their levels. The Taguchi method offers 27 experiments for this purpose.

Taguchi proposes two ways for analysis. The First is a standard approach in which the result of one-time implementation of the method or average of frequent implementations is introduced into the analysis of variance. The second approach, which Taguchi strongly recommends for frequent implementations, is the signal to noise ratio (S/N). The S/N analysis determines the most robust operating conditions against the variations of the results (Roy, 1990).

Here, the signal to noise ratio approach is used. The S/N diagram of the 27 experiments is presented in Fig. 10. Eq. (51) is used in this diagram:

$$S/N = -10Log\left(\frac{1}{n}\sum_{i=1}^{n} y^2\right) \tag{51}$$

Similarly, the Taguchi method was applied to the constructive heuristic algorithm to determine the optimal level for number of iterations and the removal probability (step 3 of the improvement phase). The selected values of these two parameters were 300 and 0.4, respectively. Here we ignore explaining more details (see Table 3).

5. Computational experiments

In this section, the solution methods and mathematical model are experimented based on 27 instances with various sizes. Four datasets have been used to perform computations and comparisons Similar to Vijayakumar et al. (2013). Table 4 shows the conditions of these sets. Table 5 shows the number of patients in the numerical examples.

The operating rooms are assumed to be active from 8 am to 6 pm. The overtime is considered from 3 pm. to 6 pm. Four types of surgical specialties including general surgery, orthopedics, otorhinolaryngology and ophthalmology are taken into account in the study.

In the literature, the majority of researchers have used the log-normal distribution to generate the duration of surgical operations (Marcon, Kharraja, Smolski, Luquet, & Viale, 2003). In this study, the durations of surgical operations are estimated via log-normal distribution (Marcon et al., 2003; Neyshabouri and Berg, 2017; Strum, May, & Vargas, 2000). Similar to Jebali et al. (2006), the durations of patients' recovery (post-anesthetic cares in the PACU) are estimated using the log-normal distribution, based on the data obtained from Marcon et al. (2003). So that, the means of surgical durations are extracted from Marcon et al. (2003) and their standard deviations are considered equal to quarter of the standard deviation of surgeries durations. The maximum deviation in surgery duration for patient p in stage j, $d\hat{u}_{pj}$, is chosen $d\hat{u}_{pj} = \alpha\sigma_{du_{pj}}$ where $\alpha \sim U(0.5, 1.5)$. All durations are first generated in minutes, and then rounded and turned into time slots. The percentage of patients in each surgical specialty is also selected randomly. Table 6 shows the details of the statistical distribution of the surgical groups and the percentage of patients in each group. The length

```
Proposed Genetic Algorithm
Input: patients' full data, data related to resources, the value of GA parameters
(npop: number of individuals in each population
pc: percentage of population to be involved in crossover
pm: percentage of population to be affected by mutation)
output: best found schedule for surgeries and the related objective function value
begin
  Step 0 (initialization)
  nc=pc*npop (number of parents for doing crossover)
  nm=pm*npop (number of mutants)
  let pop, popc and popm be the vacant population related to initial solutions, solutions affected by
  crossover and mutation
  Step 1 (Generate random initial population)
  for t=1 to npop do
      Generate a random solution (procedure 1)
      Apply feasible maker procedure on the generated solution (pop)
      evaluate the solution
  end
  Step 2 (crossover and mutation)
  Step 2.1 (crossover)
  for it=1 to Maximum Iteration do
     for i=1 to nc/2 do
        select P1 and P2 by roulette wheel selection
        apply PMX crossover on selected parent and obtain P' (child)
        Apply feasible maker procedure on P'
        evaluate the solution
        update popc
     end
     Step 2.2 (mutation)
     for i=1 to nm do
        randomly select a solution from population
        apply mutation on selected solution and obtain P''
        Apply feasible maker procedure on P''
        evaluate the solution
        update popm
     end
     Step 2.3 (forming new population)
     Merge sort and truncate pop, popc and popm
     Select the best solution: best_solution (t)=best_solution
  end
end
report the best solution of iterations
```

Fig. 6. Pseudocode of proposed genetic algorithm.

of stay in the ward and ICU is also generated using log-normal distribution, in accordance with the conditions stated by Neyshabouri and Berg (2017).

The Latest day to perform surgery, the number of operating rooms and the number of beds are generated randomly according to Vali Siar et al. (2017). Maximum desired working hours of surgeons are selected according to Van Huele and Vanhoucke (2014). In all datasets, all surgeons are available every day except for dataset 2. The availability status of surgeons in dataset 2 is presented in Table 7.

Now, the generated numerical examples are used to evaluate and compare the performance of the MILP model, proposed genetic algorithm and proposed constructive heuristic algorithm. The analysis of the results is carried out in three different parts. In the first part, the solutions are compared according to the value of the objective function and the computational time. In the second and third parts, the solutions are investigated and compared according to criteria based on hospital and patient point of views respectively. Hospital criteria include operating rooms utilization rate (U.R), the number of idle time slots (Id.T) and the number of overtime slots (OV.T). Patients' criteria include the number of operated patients (Op.), The number of patients who were operated before the latest surgery date (op. < Dt) and the number of patients who have not been operated and their latest date of surgery has passed (wait > dt).

The MILP model was coded in GAMS version 24.1.3 and the CPLEX

```
Schedule Construction
Input: patients' full data, data related to resources, Vacant structure D_SCH, List of operated
patients ope (initially vacant)
Output: Initial Schedule, obj_initial and ope.

begin
    Step 1.
    WL= Set of patients sorted randomly.
    Step 2.
    for i=1 to WL(P) do
        for d=1 to D do
            Insert patient WL(i) in all possible positions (Schedule patient WL(i) by all possible
            surgeons in all possible operating rooms in the earliest possible time slots).
            Evaluate all found schedules via fitness function
            Save the found schedules and the related evaluation values (objective values) in the
            structure D_SCH
        end
        Schedule=Find the best Schedule among found schedules existing in D_SCH based on
        evaluation value.
        Update ope.
        Update the status of all resources.
    end
    end
    Initial Schedule= Schedule.
    obj_initial = objectice value of the Initial Schedule.
report Initial Schedule, obj_initial and ope.
```

Fig. 7. Pseudocode of The first stage of CH (generation of Initial Schedule).

solver was used to solve the problem. The heuristic and metaheuristic algorithms were coded in MATLAB version R2015a. All algorithms were run with time limit of 4 h (14,400 s). In the computational time column, the problems that have been stopped before reaching the final solution (due to the time limit) are marked with ∗. The reported values for both metaheuristic and heuristic methods are the mean values of 10 implementations for each instance.

According to opinions of experts (surgeons and nurses), it is of vital importance to avoid delay in surgery and maintain patient health. Also, maximum utilization of the operating rooms, lack of idle time and overtime are considered as the second most important factors. Therefore, according to the experts' opinion weights were selected as follows: $W_t = 0.5$, $W_i = 0.25$ and $W_o = 0.25$. The computational results presented in this section are obtained considering the assumption of existing uncertainty in surgery and recovery durations.

In Table 8, the solution methods are compared in terms of the value of the objective function, and the CPU time. In the columns related to each method, the value of the objective function (Obj), the computational time (time), and the relative percentage deviation (RPD) can be seen. The RPD is calculated as below:

$$\%RPD = \frac{|Method_{sol} - Best_{sol}|}{Best_{sol}} \times 100 \tag{52}$$

In the above equation, $Method_{sol}$ is solution obtained from the method used for the problem in question and $Best_{sol}$ is the best solution to that problem.

The column titled "%Gap" related to solutions obtained by MILP denotes the gap between the current solution and the best lower bound when the problem solving process is stopped before the optimal solution is reached. In each of the two heuristic and metaheuristic methods, the values standard deviations (σ_{obj}) of the objective function values are reported. In cases where the methods are not able to find the solution before the time limit is reached, their corresponding column is marked with "–"

As Table 8 shows, using GAMS software, an optimal solution is obtained in 10 out of the total of 27 problems before the time limit is reached. Two other methods have also reached an optimal solution, which indicates their high performance. In the vast majority of other instances, the heuristic method outperforms the genetic method. More precisely, by comparing the results of the genetic algorithm and the constructive heuristic algorithm, it can be concluded that in cases where the value of the objective function is the same in two methods, the heuristic method outperforms the genetic algorithm, since this method has reached solutions by spending about one-third of the computational time spent by genetic algorithm. In other cases, the constructive heuristic method is superior to the genetic method, for the objective function value of the heuristic method, especially in large-scale problems, is better than the objective function of genetic method, and this method can reach the solution within a much shorter period of time. Overall, it can be said that constructive heuristic method has outperformed the MILP and genetic methods in terms of the objective function value and computational time. Also, both proposed algorithms have very small standard deviations, and in most of these problems their standard deviation is about zero, indicating the high-quality of the proposed methods.

Fig. 11 shows the comparison between the objective function values in different problems. As seen, in small size problems, there is no difference between methods, but with increase in the size of the instances, MILP loses its efficiency. Among two other methods, the constructive heuristic method has a better performance. This difference is more significant in larger scale instances (instances 19–27).

Table 9, compares the solution methods with consideration of the hospital criteria. In small size problems, for which optimal or relatively optimal solutions are provided using MILP (according to Table 8), no difference is observed between the results of the genetic algorithm and the proposed constructive technique. As the size of the instances grows,

```
Improving
Input: patients' full data, data related to resources , vector re (list of removed patients-initially
vacant), Initial Schedul, obj_initial, ope (these items come from Phase 1), pr (removal probability),
re (list of removed surgeries that is initially vacant)
output: Improved schedule
begin
Schedule= Initial Schedule.
obj = obj_initial.
  Step 1.
  for k=1 to P do
    if any time slot of surgery k is scheduled in overtime do
      Schedule_new = remove surgery k from schedule, obj_new =Evaluate the obtained schedule.
    end
    if obj_initial > obj_new
      Schedule= Schedule_new, obj = obj_new.
      Update the status of recourses and Update ope.
    end
  Step 2.
  for k=1 to P do
    If patient k is not operated (doesn't exist in list ope) do
      Schedule the patient k if possible.
      obj_new = Evaluate the obtained schedule.
    end
    if obj_initial > obj_new do
      Schedule= Schedule_new, obj_initial = obj_new.
      Update the status of recourses and Update ope.
    end
  end
  Step 3.
  for i=1 to (p_r*|ope|) do
    randomly select a surgery (which exists in the schedule)
    remove the selected surgery from schedule.
    put the removed surgery in the list re.
  end
  for k=1 to |re| do
    Schedule patient k and obtain Schedule_new.
    obj_new = evaluate the obtained schedule.
    if obj_initial > obj_new do
      Schedule= Schedule_new, obj_initial = obj_new.
      Update the status of recourses and Update ope.
    end
  end
end
imp_schedule= Schedule.
report imp_schedule.
```

Fig. 8. Pseudocode of The first phase of CH (Improving).

the proposed heuristic and metaheuristic methods offer better performance in comparison with MILP. Meanwhile, the constructive heuristic method has a better performance with about 12% lower idle time and 80% lower overtime compared with the genetic method.

The results obtained from the comparison of the solution methods based on the patients' criteria are also reported in Table 10. In most cases, the number of operated patients in the proposed heuristic method is greater, and the number of patients whose surgery is delayed is lower. In large-scale instances, the differences between the genetic algorithm and constructive heuristic method have become more significant. For example, the average number of patients in the heuristic method is 20% more than the genetic method.

- Pairwise statistical comparisons of solution methods

In order to make the comparisons statistically convincing and

Constructive heuristic (CH)

Input: patients' full data, data related to resources, the value of CH parameters, vacant structure *ANS*
output: best found schedule for surgeries and the related objective function value
begin

For *it*=1:*Maximum Iteration* **do**
  Phase 1 (producing initial schedule)
  Phase 2 (improvement)
  Step 1
  Step 2
  Step 3
  $Schedule_{it}$ = obtained schedule.
  Obj_{it} = evaluate the $Schedule_{it}$ $Schedule_{it}$.
  Save the $Schedule_{it}$ and Obj_{it} in *ANS.*
end
Find the best found schedule.
report best fond schedule and the related objective function value

Fig. 9. Pseudocode of the general procedure of the proposed CH algorithm.

Table 2
Parameters of genetic algorithm and their levels.

Level/ parameter	Number of iterations	Population	Probability of crossover	Probability of mutation	Mutation rate
1	100	50	0.6	0.05	0.4
2	200	100	0.7	0.1	0.5
3	300	200	0.8	0.15	0.6

Table 3
Selected levels for genetic parameters based on the results of parameter setting.

Parameter	Number of iterations	Population	Probability of crossover	Probability of mutation	Mutation rate
Symbol	MAxit	Npop	Pc	pm	P_{mut}
Value	200	100	0.8	0.15	0.4

comparing the solution methods more accurately, in this section, statistical tests are performed based on different criteria using SPSS (version 24) software. Using the Kolmogorov-Smirnov test at the significant level of 0.05, it was found that the results presented in the previous tables do not follow normal distribution, so here the nonparametric Wilcoxon signed rank test is applied at the significance level of 0.05. In Table 11 the values of asymptotic significance (A.S.) level and the Z statistic are reported. In cases where the significance level is less than 0.05, the difference between the two methods is statistically significant.

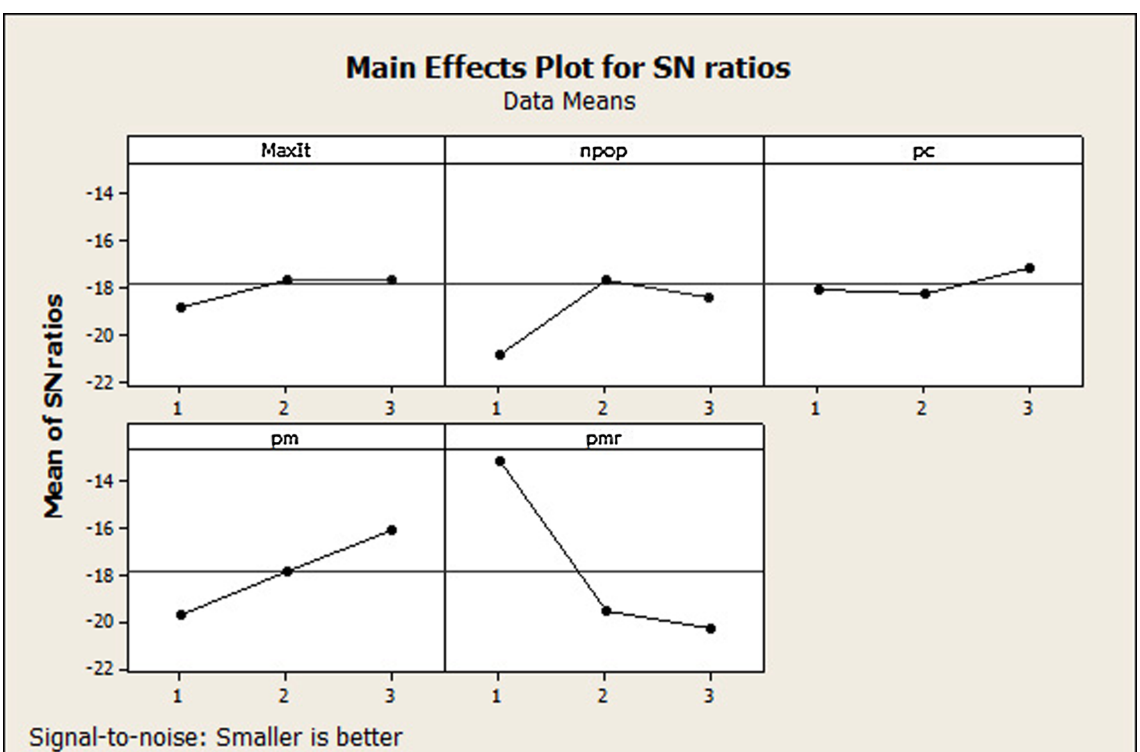

Fig. 10. S/N ratio diagram related to the parameters of the genetic algorithm.

Table 4
Datasets used for computational analysis.

Parameters/Dataset	1	2	3	4
Total number of allowed time slots for performing surgery	36	36	36	36
Number of allowed time slots for overtime	12	12	12	12
Length of each time slot	20 min	20 min	20 min	20 min
Number of operating rooms	5	5	5	8
Number of surgeons	5	5	5	8
Number of anesthesiologist	5	5	5	8
Number of PHU beds	5	5	5	8
Number of PACU beds	6	7	8	7
Number of ICU beds	4	6	5	7
Number of ward beds	16	18	20	22
Number of surgical specialties	4	4	4	4
Number of equipment types	4	4	4	4
Number of equipment of each type	10	8	10	18
Number of scrub/circulator/anesthetist nurses	7/7/7	7/7/7	7/7/7	9/9/9
Number of days of planning horizon	2	3	6	6
Maximum allowed working time slots for surgeons	48	72	144	288
Number of General/Orthopedic/ear, nose, and throat (ENT)/ophthalmology surgeons	2/1/1/1	2/1/1/1	2/1/1/1	3/2/2/1

Table 5
Numerical examples used for computational analysis.

Dataset	Problem no.	Number of patients
1	1–5	5/10/15/20/30
2	6–12	5/10/15/20/30/40/50
3	13–22	5/10/15/20/30/40/50/60/70/80
4	22–27	90/100/110/120/130

Table 6
Details of the statistical distribution of the surgery duration of different surgical groups.

Surgical Specialty	Average of surgery duration (minutes)	Standard deviation of surgery duration (minutes)	Percentage of patients
Orthopedics	120	30	30.46%
Ophthalmology	60	15	90.00%
General	180	60	38.00%
ENT	45	30	22.54%

Table 7
Access to surgeons during scheduling days in dataset 2.

Surgeon/day	1 (time slots)	2 (time slots)	3 (time slots)
1 General	1–36	13–36	1–24
2 General	1–24	1–36	7–36
3 Orthopedics	1–36	1–36	10–36
4 ENT	7–24	1–36	1–36
5 Ophthalmology	Absence	1–36	13–24

According to Table 11 in the patients' criteria, there is no statistically significant difference between the MILP and the other two methods. But in all of the patients' criteria, the proposed constructive heuristic method outperforms the genetic method. In criteria of operating room utilization and overtime, there is no significant difference between the genetic method and the MILP, while the heuristic method provides better results compared to other methods. Based on the objective function and its standard deviation the heuristic method shows better performance compared with genetic algorithm. Finally, based on the computational time, heuristic and meta-heuristic outperform the MILP. Also, the proposed heuristic method reaches the solution in a shorter CPU time compared to the genetic method. According to the results achieved, the proposed constructive heuristic method has the best performance.

The comparison between the objective function values of the methods in deterministic and uncertain conditions for different problems is presented in Fig. 12 ("U" shows the solution method has been run uncertainty). With a closer look at this figure, it is found that the value of the objective function in uncertain mode is relatively less (better) than the deterministic mode.

In this study, computations were performed assuming deterministic length of stay in the ICU and ward. However, the operating room scheduling can be affected by uncertainty in the length of stay, because the capacity of ICU and ward is limited. Due to the stated reasons, a number of simulations were implemented to test the robustness of the schedules. 100 simulations were performed for each numerical instance, taking into account the variability in the patients' length of stay. The variability of the length of stay is determined based on the log-normal distribution with consideration of mean length of stay and standard deviations of 5%, 10%, 15% and 20%. Robustness is assessed according to idle time, operating room utilization rate, the number of operated patients, and the number of patients who have not been operated and their surgery is delayed. The reason for choosing these criteria for robustness assessment is that when the ICU or ward is facing challenges associated with resource constraints, less surgical operations are performed and the utilization rate declines, therefore a number of patients may also face with delay in their surgeries. The results are presented in Table 12. In these computations, the durations of surgery and recovery are also considered uncertain.

As can be seen, the criteria are not influenced by the uncertainties in the length of stay, or are insignificantly influenced. Base on the idle time, utilization rate and the number of operated patients, impacts are also insignificant. The number of patients who have not been operated and their surgeries are delayed has not been changed. Therefore, the solutions are robust against the uncertainty in the length of stay.

6. Case study

In this section, scheduling performed in a hospital is compared to the scheduling obtained from the proposed methods in order to measure the performance of proposed methods in real life situations. There are four surgery specialties in the hospital including general surgeries, orthopedics, obstetrics and gynecology and ophthalmology. Operating room Scheduling is performed by the head nurse usually on a weekly basis. The surgical data are available to the operating theater at least 48 h before the first day of week (Saturday) and surgeries are scheduled on a weekly basis (6 days) based on the latest surgery date determined by the specialist surgeon. Due to the traffic in surgeries, the remaining surgeries in the waiting list or the surgeries recorded in the waiting list during the planning horizon will be scheduled in the coming weeks

Data for a 2-week period in October of 2016 was available to us which is summarized in Table 13. As already mentioned, only the elective surgeries are taken into account.

Table 14 and Figs. 13 and 14 compare the performance of proposed scheduling methods (PM) and scheduling of hospital (HS). According to the results, in the first week, all three methods have reached same and high-quality solutions. In the second week, the mathematical model and the constructive heuristic method have reached optimal solution and have outperformed the genetic algorithm and hospital schedule. The analyses and computations are done with considering uncertainty in surgery and recovery durations. The idle time of the proposed methods is 19% and 14% lower than the idle time of the hospital schedule in the first and second weeks respectively. Also, overtime of the surgery department of the hospital can decrease about 73% in the first week and about 47% in the second week via implementing the proposed methods. According to the results, the constructive heuristic method is the most

Table 8
Comparing solution methods based on objective function and CPU time.

Instance	MILP (Cplex)				GA				CH			
	Obj.	%Gap	Time	% RPD	Obj.	Time	$\sigma_{Obj.}$	% RPD	Obj.	Time	$\sigma_{Obj.}$	% RPD
1	0.19	0.00	2.65	0.00	0.19	0.70	0.00	0.00	0.19	0.11	0.00	0.00
2	0.17	0.00	31.13	0.00	0.17	1.16	0.00	0.00	0.17	0.13	0.00	0.00
3	0.12	5.31	*	0.00	0.12	204.09	0.00	0.00	0.12	25.30	0.00	0.00
4	0.09	5.88	*	0.00	0.09	274.06	0.00	0.00	0.09	16.66	0.00	0.00
5	0.12	54.24	*	50.00	0.08	245.25	0.01	33.33	0.06	71.24	0.00	0.00
6	0.21	0.00	3.91	0.00	0.21	0.53	0.00	0.00	0.21	0.42	0.00	0.00
7	0.17	0.00	31.05	0.00	0.17	1.08	0.00	0.00	0.17	0.53	0.00	0.00
8	0.14	0.00	268.57	0.00	0.14	7.98	0.00	0.00	0.14	2.57	0.00	0.00
9	0.12	4.60	*	0.00	0.12	95.55	0.00	0.00	0.12	61.15	0.00	0.00
10	0.12	15.51	*	9.00	0.12	1443.27	0.00	9.09	0.11	110.82	0.00	0.00
11	–	–	*	–	0.15	1976.23	0.01	36.36	0.11	116.07	0.00	0.00
12	–	–	*	–	0.17	917.52	0.02	21.43	14	100.15	0.00	0.00
13	0.23	0.00	5.12	0.00	0.23	0.48	0.00	0.00	0.23	0.15	0.00	0.00
14	0.21	0.00	118.13	0.00	0.21	0.76	0.00	0.00	0.21	0.34	0.00	0.00
15	0.21	0.00	225.14	0.00	0.21	1.08	0.00	0.00	0.21	0.57	0.00	0.00
16	0.18	0.00	4326.45	0.00	0.18	1.51	0.00	0.00	0.18	0.95	0.00	0.00
17	0.14	0.00	10802.08	0.00	0.14	74.47	0.00	0.00	0.14	6.32	0.00	0.00
18	–	–	*	–	0.13	186.23	0.00	0.00	0.12	131.82	0.00	0.00
19	–	–	*	–	0.11	736.10	0.00	22.22	0.09	222.26	0.00	0.00
20	–	–	*	–	0.12	1058.74	0.01	20.00	0.10	626.24	0.00	0.00
21	–	–	*	–	0.13	1332.08	0.00	85.71	0.07	462.64	0.00	0.00
22	–	–	*	–	0.13	1515.08	0.01	62.50	0.08	664.43	0.00	0.00
23	–	–	*	–	0.14	6218.13	0.01	27.27	0.11	2219.25	0.00	0.00
24	–	–	*	–	0.14	6839.34	0.01	55.56	0.09	3015.35	0.00	0.00
25	–	–	*	–	0.14	4662.51	0.01	75.00	0.08	1475.65	0.00	0.00
26	–	–	*	–	0.14	8373.87	0.00	75.00	0.08	3844.40	0.00	0.00
27	–	–	*	–	0.16	8509.34	0.01	77.78	0.09	3901.20	0.00	0.00
Average	0.16	9.29	6054.28	3.93	0.15	1654.74	0.00	22.27	0.13	632.47	0.00	0.00

efficient one, for it reaches high-quality solutions in less than 30 s. Summing up, the results obtained using the proposed methods are of much higher quality compared to the actual schedules provided by the hospital.

In Table 15, the mean duration of surgery and recovery, and also expected duration of length of stay have been changed up to 5%, 10%, 15%, and 20% and the values of performance criteria have been reported. The last row of the table shows the change in the objective function value relative to the objective function values presented in Table 14. Since the constructive heuristic method outperforms other methods, this method is applied here. Note that the computations are done under the uncertainty of duration of surgery and recovery.

The results show that despite the increase in mean duration of surgery, recovery and length of stay, the schedules obtained from the proposed method are still much better than hospital schedule. According to the results, the operating room utilization rate has improved, and with increasing the percentage of durations has led to an increase in overtime. In terms of patients' criteria, all patients are scheduled before the latest surgery date, and only when the durations increase by 15% and 20%, one and two surgeries are delayed respectively, which can be compensated through overtime. Therefore, even with increasing the mean durations of surgeries under uncertainty, and the expected durations of lengths of stay, planning and scheduling based on the proposed methods prove to be very efficient and quite superior to hospital schedule.

7. Conclusion

In this paper, the integrated operating room planning and scheduling is investigated for elective patients under uncertainty. In addition to operating rooms, the PHU, recovery (PACU), ICU and ward were taken into account. Constraints related to human resources, equipment and beds were taken into consideration. A linear mixed integer programming model was presented to formulate the problem. The intended objectives were to minimize the tardiness of surgeries which is a criterion based on patients point of view and minimize idle time and overtime as criteria based on hospital point of view. The durations of surgery and recovery are considered uncertain and a robust optimization approach is used to manage uncertainty. Due to the high complexity of the model, a meta-heuristic method based on the genetic

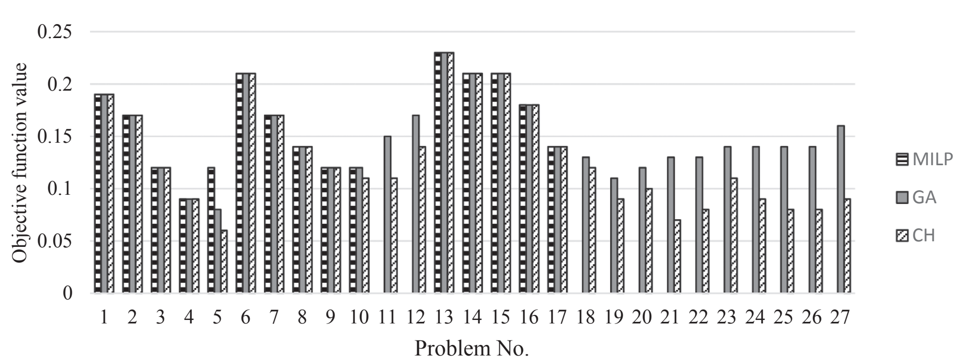

Fig. 11. Comparing solution methods based on objective function (under uncertainty).

Table 9
Comparing solution methods based on hospital's criteria.

	MILP (Cplex)			GA			CH		
Instance	U.R	Id.T	Ov.T	U.R	Id.T	Ov.T	U.R	Id.T	Ov.T
1	0.24	183	0	0.24	182	0	0.24	182	0
2	0.34	159	0	0.34	159	0	0.34	159	0
3	0.54	111	0	0.54	111	1	0.54	111	1
4	0.65	85	0	0.66	81	3	0.66	82	1
5	0.54	110	1	0.71	71	1	0.75	60	0
6	0.18	297	0	0.18	297	0	0.18	297	0
7	0.31	250	0	0.31	250	0	0.31	250	0
8	0.44	203	0	0.43	204	1	0.44	203	0
9	0.52	174	0	0.54	167	1	0.54	167	1
10	0.52	174	0	0.58	151	8	0.57	153	0
11	–	–	–	0.67	120	36	0.65	127	3
12	–	–	–	0.65	126	0	0.67	120	5
13	0.08	660	0	0.08	660	0	0.08	660	0
14	0.15	610	0	0.15	610	0	0.15	610	0
15	0.17	596	0	0.17	596	0	0.17	596	0
16	0.28	518	2	0.28	518	0	0.28	518	0
17	0.44	406	0	0.44	406	0	0.44	406	0
18	–	–	–	0.49	366	3	0.55	326	1
19	–	–	–	0.61	278	11	0.65	252	1
20	–	–	–	0.57	310	7	0.64	260	6
21	–	–	–	0.54	330	14	0.76	172	3
22	–	–	–	0.61	281	13	0.74	186	5
23	–	–	–	0.55	559	5	0.60	461	3
24	–	–	–	0.58	492	67	0.67	384	1
25	–	–	–	0.58	484	23	0.74	306	4
26	–	–	–	0.61	452	35	0.75	292	5
27	–	–	–	0.55	532	35	0.71	348	9
Average	0.61	308.82	0.24	0.63	306.15	9.44	0.67	272.04	1.48

algorithm and a constructive heuristic method are proposed to solve medium and large-scale problems. Comparisons and computations were performed based on the objective function value, computational time, and patient and hospital criteria. The results indicate that heuristic and metaheuristic methods are very efficient in obtaining high-quality solutions. Comparison of the proposed constructive heuristic method and the genetic-based metaheuristic method showed that the heuristic method has a better performance and the objective function value in this method is about 19% better than the objective function value obtained from the genetic algorithm.

In the case study, the performance of the proposed methods was compared to the actual schedule of a hospital. The results indicated that in the schedule provided by the proposed methods, overtime is one-sixth of the overtime of the hospital schedules. In addition, the idle time

Table 11
Wilcoxon signed-rank test for MILP, CH and GA based on performance measures.

Performance measure		CH-MILP	GA-MILP	CH–GA
Number of operated patients	Z	−1.86	−1.84	−2.59
	A.S.	0.06	0.07	0.01
Number of operated patients without tardiness	Z	−1.86	−1.84	−2.68
	A.S.	0.06	0.07	0.01
The number of patients who have not undergone surgery and are tardy	Z	0.00	0.00	−2.41
	A.S.	1.00	1.00	0.02
%Utilization of operating rooms	Z	−2.02	−1.89	−3.13
	A.S.	0.04	0.06	0.02
Overtime	Z	0.00	−1.54	−3.52
	A.S.	1.00	0.11	0.00
Objective function	Z	−1.34	−1.00	−3.31
	A.S.	0.18	0.32	0.01
SD of objective function	Z	–	–	−2.89
	A.S.	–	–	0.04
Computational time	Z	−3.52	−3.52	−4.54
	A.S.	0.00	0.00	0.00

Table 10
Comparing solution methods based on patients' criteria.

	MILP (Cplex)			GA			CH		
Instance	Op.	Op. < dt	Wait > dt	Op.	Op. < dt	Wait > dt	Op.	Op. < dt	Wait > dt
1	5	5	0	5	5	0	5	5	0
2	10	10	0	10	10	0	10	10	0
3	12	12	0	12	12	0	12	12	0
4	15	15	0	17	17	0	16	16	0
5	19	19	0	20	20	0	21	21	0
6	5	5	0	5	5	0	5	5	0
7	10	10	0	10	10	0	10	10	0
8	15	15	0	15	15	0	15	15	0
9	19	19	0	20	20	0	20	20	0
10	22	22	0	26	26	0	24	24	0
11	–	–	–	31	30	3	30	29	4
12	–	–	–	35	34	9	32	32	10
13	5	5	0	5	5	0	5	5	0
14	10	10	0	10	10	0	10	10	0
15	15	15	0	15	15	0	15	15	0
16	20	20	0	20	20	0	20	20	0
17	30	30	0	30	30	0	30	30	0
18	–	–	–	34	33	1	39	38	1
19	–	–	–	48	46	0	50	49	0
20	–	–	–	47	47	3	54	53	0
21	–	–	–	62	60	2	68	67	1
22	–	–	–	53	49	7	66	63	2
23	–	–	–	67	63	4	80	78	2
24	–	–	–	90	89	3	96	95	1
25	–	–	–	88	84	7	99	95	4
26	–	–	–	89	86	6	105	93	3
27	–	–	–	88	80	11	111	107	6
Average	14.78	14.78	0	23.31	22.00	2.54	28.14	26.77	1.64

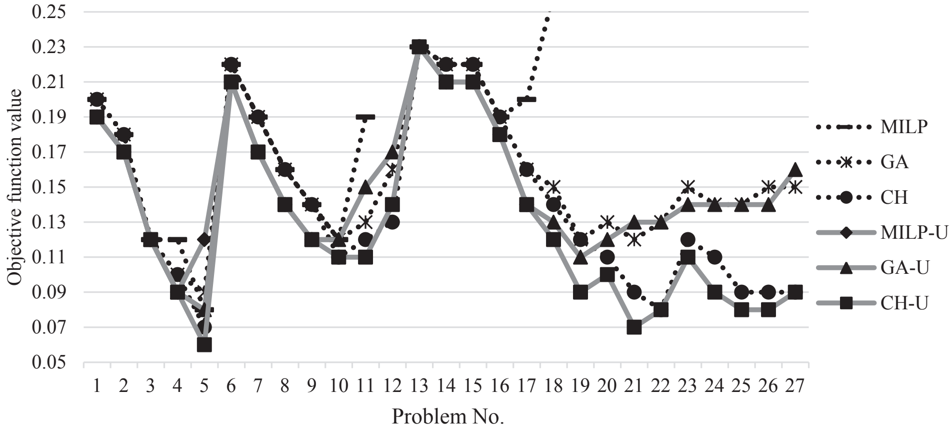

Fig. 12. Comparing solution methods based on objective function in deterministic and uncertain modes.

Table 12
Investigating the robustness of solutions using simulation.

Standard deviation	Idle time	Overtime	Operated patients	Tardy patients
0%	272.04	0.67	28.64	1.14
5%	272.04	0.67	28.64	1.14
10%	272.91	0.67	27.60	1.14
15%	272.91	0.67	27.48	1.14
20%	278.45	0.66	26.94	1.14

Table 13
Summary of data collected from the surgery department of the hospital.

Number of patients in week 1	51
Number of patients in week 2	45
Number of days in the planning horizon	6 (one week)
Number of operating rooms	3
Number of surgical specialties	4
Number of surgeons	11
Number of nurses	9
Number of critical equipment types	4
Number of equipment in each type	3
Length of each time slot	20 min
Total number of time slots	27 (8:00 AM to 5 PM)
Number of overtime time slots	9 (2:00 PM to 5 PM)
Number of PHU beds	3
Number of PACU beds	2
Number of ICU beds	4
Number of ward beds	33
Mean surgery duration	0.5–5 h per case
The maximum deviation in surgery duration (from the mean)	10–30% per case
Mean duration of recovery	0.5–2 h per case
The maximum deviation in recovery duration (from the mean)	5–25% per case
Expected duration of staying in PHU	10–30 min per case
Expected duration of staying in ICU/ward	0–2 days/0–5 days per case

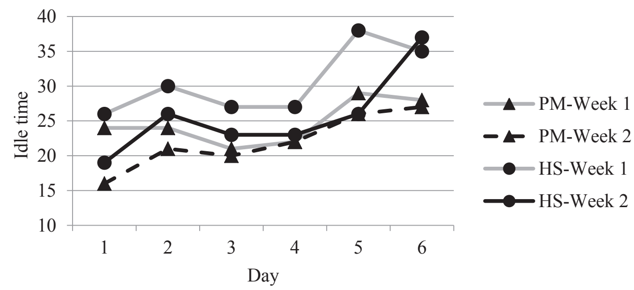

Fig. 13. Comparing the proposed methods and the hospital schedule based on operating rooms idle time.

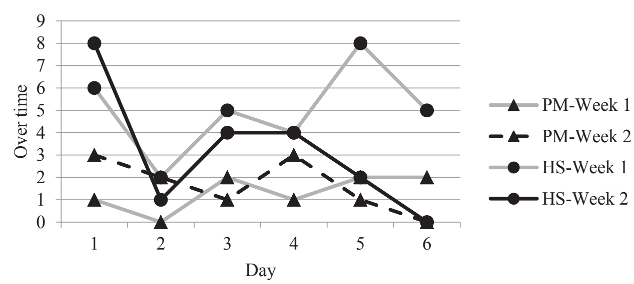

Fig. 14. Comparing the proposed methods and the hospital schedule based on operating rooms overtime.

in the proposed methods is about 25% better.

The problem studied in this paper is based on the assumption that the resources used for elective and emergency patients are independent. Future research can be done based on the assumption that the same resources are used for these two groups. Developing exact solution

Table 14
Comparing the proposed methods and the hospital schedule.

	MILP			GA			CH			Hospital schedule		
Measure	Ov.T	Id.T	U.R	Ov.T	Id.T	U.R	Ov.T	Id.T	U.R	Ov.T	Id.T	U.R
Week 1	8	148	0.54	14	154	0.52	8	148	0.54	30	183	0.44
Week 2	10	132	0.59	13	135	0.58	10	132	0.59	19	154	0.54
Measure	> dt	< dt	Op.	> dt	< dt	Op.	> dt	< dt	Op.	Op.	< dt	Op.
Week 1	0	51	51	0	51	51	0	51	51	0	51	51
Week 2	0	45	45	0	45	45	0	45	45	0	45	45
Measure	% RPD_{obj}	Time	Obj.	% RPD_{obj}	Time	Obj.	% RPD_{obj}	Time	Obj.	% RPD_{obj}	Time	Obj.
Week 1	0.00	*	0.13	7.69	308.21	0.14	0.00	9.14	0.13	46.15	–	0.19
Week 2	0.00	821.12	0.12	0.00	314.54	0.12	0.00	23.36	0.12	25.00	–	0.15

Table 15
Evaluation of changes in performance criteria with changing durations LOS in case study.

Variation%	5%		10%		15%		20%	
Measure	Week 1	Week 2	Week 1	Week 2	Week 1	Week 2	Week 1	Week 2
Id.T	146	130	143	121	133	119	116	108
U.R	0.55	0.60	0.56	0.61	0.59	0.63	0.64	0.67
Ov.T	8	10	9	15	12	23	17	28
Op.	51	45	51	45	51	45	51	45
< dt	51	45	51	45	51	44	50	43
> dt	0	0	0	0	0	0	0	0
Obj.	0.13	0.12	0.12	0.12	0.12	0.13	0.12	0.13
%∇ Obj.	0.00	0.00	−8.33	0.00	−8.33	8.33	−8.33	8.33

methods for obtaining optimal solutions for different problems can be a great idea for the future studies. Finally, taking into account other important performance criteria in the literature, including patient waiting time and financial objectives, can be considered in future researches.

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