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Brief paper

Resilient adaptive and H_∞ controls of multi-agent systems under sensor and actuator faults[☆]

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ABSTRACT

Resilience of multi-agent systems (MAS) reflects their capability to maintain normal operation, at a prescribed level in the presence of unintended faults. In this paper, we investigate resilient control of MAS under faults on sensors and actuators. We propose four resilient state feedback based leader–follower tracking protocols. For the case of sensor faults, we develop an adaptive compensation protocol and an H_∞ control protocol. For the case of simultaneous sensor and actuator faults, we further propose an enhanced adaptive compensation protocol and an enhanced H_∞ control protocol. We show the duality between the adaptive compensation protocols and the H_∞ control protocols. For adaptive compensation protocols, faults on sensors and actuators are rejected by using local adaptive sensor and actuator compensators, respectively. Moreover, by employing a static output-feedback design technique, we propose H_∞ control protocols that guarantee bounded L_2 gains of certain errors in terms of the L_2 norms of fault signals. This further allows us to prove resilience even if sensor faults are unbounded. Finally, simulation studies validate the effectiveness of the proposed protocols.

1. Introduction

The last decade has witnessed significant development of cooperative control techniques for interconnected multi-agent systems (MAS) (see Olfati-Saber, Fax, and Murray (2007), Ren, Beard, and Atkins (2007), Ren and Cao (2010) and Lewis, Zhang, Hengster-Movric, and Das (2013) for surveys). Therein, a distributed controller is designed for each agent locally, based on information only about that agent and its neighbors. The benefits of such distributed architectures over standard centralized controllers include greater efficiency, flexibility, and scalability to larger networks. Hence, distributed controllers have emerged in many engineering applications including power systems (Bidram, Lewis, & Davoudi, 2014; Robbins & Hadjicostis, 2013), robotic networks (Bullo, Cortes, & Martinez, 2009; Qu, 2009; Ren & Beard, 2008), and sensor networks (Cortés, Martınez, Karatas, & Bullo, 2004; Ogren, Fiorelli, & Leonard, 2004). Since the distributed information flow lacks a centralized feedback mechanism, the activity of each agent cannot be effectively monitored and verified. This makes distributed protocols particularly susceptible to adverse faults that are injected to the sensors and/or actuators of agent, and can propagate through the network. Therefore, it remains a challenge to provide resilience for the distributed control of MAS.

To address the resilient MAS problem, several attack detection and isolation approaches have been proposed in the literature (see Fawzi, Tabuada, and Diggavi (2014), LeBlanc, Zhang, Sundaram, and Koutsoukos (2012, 2013), Mo, Chabukswar, and Sinopoli (2014), Pasqualetti, Bicchi, and Bullo (2012), Pasqualetti, Dörfler, and Bullo (2013), Xu and Zhu (2015), Zeng and Chow (2014), Zhu and Başar (2015), Zhu and Martínez (2013) and Zeng, Zhang, and Chow (2015)). Despite good performances, these approaches usually make specific assumptions on the graph topology and/or the fraction of misbehaving agents.

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Such limiting assumptions are removed for a network of homogeneous systems in De La Torre, Yucelen, and Peterson (2014), where a local state emulator adaptively mitigates actuator faults. In addition, adaptive backstepping control approaches have been used to handle the actuator faults (Chen, Xie, Lewis, Xie, & Davoudi, 2018; Wang & Wen, 2011; Wang, Wen & Guo, 2016). Later, results are reported to achieve an infinite-actuator fault control (Wang, Wen & Lin, 2017), a finite-time control (Wang, Song, Krstic & Wen, 2016), event-triggered input control (Xing, Wen, Liu, Su, & Cai, 2017), multi-vehicle system (Liu, Ma, Lewis, & Wan, 2018) and input quantization control (Li & Yang, 2016; Wang, Wen, Lin & Wang, 2017). Assuming the disturbances and actuator faults unknown and bounded, a decentralized adaptive fault-tolerant control is given in Xie and Yang (2017). Note that majority of existing adaptive resilient protocols focus on actuator faults. In the presence of sensor faults, measured states will no longer be the same as actual states, rendering the above-mentioned adaptive controls impractical. Remedies are provided through adaptive stabilization control (Yucelen, Haddad, & Feron, 2016) and adaptive leaderless control (Arabi, Yucelen, & Haddad, 2016). They, however, restrict the sensor fault types to certain bounded faults and/or types with known structures. More importantly, simultaneous sensor and actuator faults are not addressed in a unified platform.

This paper aims to design resilient distributed protocols subject to both sensor and actuator faults. We design four resilient leader–follower tracking control protocols, which can be separated into two main categories — adaptive control and H_∞ control. The salient contributions of this paper are stated as follows.

• For the cases where there are only sensor faults, we design two resilient protocols, namely an adaptive compensation protocol and an H_∞ control protocol, that are in some sense dual to one another. In comparison to the adaptive control of Theorem 1, it is seen that Assumption 3 required in H_∞ control is weaker than Assumption 2.

• For simultaneous sensor and actuator faults, we also design an adaptive compensation protocol and an H_∞ control protocol. An additional actuator disturbance compensator must be added for the adaptive control protocol. On the other hand, for the H_∞ control protocol, the actuator fault is not adaptively compensated but modeled as a part of system disturbances.

• Even though the adversity of the sensor fault is physically different from that of the actuator fault, our results show that both faults can be handled within the same adaptive framework where the fault is rejected by a local compensator (see Theorems 1 and 3).

• Compared with our adaptive protocols, H_∞ protocols here (see Theorems 2 and 4) allow unbounded faults on sensors. Moreover, the use of the static output-feedback design technique in H_∞ control protocols allows us to bound the L_2 gains of the control errors in terms of the L_2 norms of the fault signals.

The rest of this paper is organized as follows. In Section 2, we provide notation and preliminaries, and define two types of fault control problems for the resilient design. In Section 3, we present two resilient solutions to the first problem based on adaptive compensation control and H_∞ control. In Section 4, we provide two additional protocols to the second problem, and extend the results in Section 3 to a more general case with simultaneous faults on both sensors and actuators. In Section 5, simulation studies validate the effectiveness of the proposed protocols. In Section 6, we conclude the results.

2. Preliminaries and problem formulation

In this section, we introduce notations and formulate the sensor and actuator fault problems for leader–follower tracking control.

2.1. Preliminaries

The symbol $\otimes$ denotes the Kronecker product. $[x_{ij}]$ is a matrix with x_{ij} an entry in the ith row and jth column. $diag\{x_i\}$ is a diagonal matrix with a vector x_i on the main diagonal. $X > 0$ denotes that a matrix X is positive-definite. For $\lambda_i \in \mathbb{C}$ and $X \in \mathbb{R}^{n\times n}$, let λ_i denote an eigenvalue of X, and $Re(\lambda_i)(X)$ denote the real part of λ_i with $i = 1, 2, \ldots, n$. $\delta_{\min}(X)$ and $\delta_{\max}(X)$ are minimum and maximum singular values of X, respectively. $\|\cdot\|$ denotes the norm.

Consider a class of directed graphs described as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ denotes a set of nodes, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes a set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes an adjacency matrix. The information flow in the graph $\mathcal{G}$ is denoted by a weight a_{ij} and an edge (v_j, v_i) satisfying $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. Here, we assume that no repeated edges or no self-loops are allowed in $\mathcal{G}$. Define $\mathcal{N}_i = \{j|(v_j, v_i) \in \mathcal{E}\}$ as a set of neighbors of node i, and $H = diag\{h_i\} \in \mathbb{R}^{N\times N}$ as an in-degree matrix with $h_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. Hence, the Laplacian matrix is given as $L = H - \mathcal{A}$. A direct path from node i to node j is captured by a sequence of successive edges satisfying $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. A graph is said to have a spanning tree, if there exists a directed path from a node to every other node. If the leader node is a neighbor of node i, then an edge (v_0, v_i) exists with a positive weighting gain g_i. Considering N followers in the graph, one has a pinning matrix $G = diag\{g_i\} \in \mathbb{R}^{N\times N}$. Throughout this paper, the following standard assumption of the graph topology in distributed control of MAS holds.

Assumption 1. The directed graph $\mathcal{G}$ contains a spanning tree with the leader as its root.

2.2. Problem formulation

Consider a group of N agents with identical dynamics

$$\dot{x}_i = Ax_i + B\bar{u}_i, \quad y_i = \bar{x}_i, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the system state, $\bar{u}_i \in \mathbb{R}^m$ is the system input, and $y_i \in \mathbb{R}^p$ is the measured output for $i = 1, 2, \ldots, N$. For convenience, we omit the time argument t throughout the paper. If necessary, however, we explicitly write the time argument t. (A, B) is assumed to be stabilizable. Even though full state feedback is assumed throughout this paper, only the corrupted states, not the actual ones, are available for the resilient control design. The dynamics of the leader agent is given by

$$\dot{x}_0 = Ax_0, \tag{2}$$

where $x_0 \in \mathbb{R}^n$ is the system state. The leader agent (2) can be considered as an exosystem or a command generator, which generates the desired target trajectory to be followed by all N agents (1). Note that the agents given in (1) and (2) are connected by a distributed communication graph, and the leader agent (2) can be observed by a small group of the agents (1).

In this paper, the system input $\bar{u}_i$ and output y_i are under unknown actuator and sensor faults, respectively. We describe the sensor and actuator faults as

$$\bar{x}_i = x_i + \delta_i^s, \tag{3}$$

$$\bar{u}_i = u_i + \delta_i^a, \tag{4}$$

where δ_i^s and δ_i^a denote unknown faults caused in the sensor and actuator channels, respectively. That is, the actual values of x_i and δ_i^s are unknown and one can only measure the corrupted state $\bar{x}_i$. Likewise, the uncorrupted control u_i cannot be applied to the system. Only the corrupted control $\bar{u}_i$ enters the dynamics (1).

To highlight the problems caused by sensor and actuator faults (3), (4), consider a standard local protocol for the leader–follower tracking (Zhang, Lewis, & Das, 2011)

$$u_i = c_s K_s(\sum_{j\in\mathcal{N}_i} a_{ij}(\bar{x}_j - \bar{x}_i) + g_i(x_0 - \bar{x}_i)), \tag{5}$$

where c_s and K_s are design gains. Under faults (3), (4), applying u_i (5) to MAS (1) yields $\dot{x} = (I_N \otimes A)x - c(L+G)\otimes BK_s(x - \underline{x}_0) + (I_N \otimes B)W$, where $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$, $\underline{x}_0 = I_N \otimes x_0$, and $W = [w_1^T, w_2^T, \ldots, w_N^T]^T$ with $w_i = \delta_i^a + c_s K_s(\sum_{j\in\mathcal{N}_i} a_{ij}(\delta_i^s - \delta_j^s) + g_i\delta_i^s)$. It is shown in Yucelen et al. (2016) and Arabi et al. (2016) that under faults δ_i^s and δ_i^a, the leader–follower tracking is generally not attained, and x_i may even be unbounded. In this paper, we show how to achieve leader–follower tracking if δ_i^s and δ_i^a are not zero, or even unbounded.

To evaluate the protocol to be given in this paper, we give the following stability definition.

Definition 1 (*Definition 4.6, Khalil (2002)*). The signal $z(t)$ is said to be uniformly ultimately bounded (UUB) with the ultimate bound b, if given positive constants b and c, for every $a \in (0, c)$, there exists $T(a, b)$, independent of t_0, such that $\|z(t_0)\| \le a \Rightarrow \|z(t)\| \le b, \forall t \ge t_0 + T$.

Two types of fault control problems for the state synchronization of MAS are defined as follows.

Problem 1. Under the sensor fault (3), the leader–follower tracking problem is to design distributed control protocols u_i for all the agents (1) in $\mathcal{G}$, such that the error $\mu_i = x_i - x_0$ is UUB.

Problem 2. Under the sensor fault (3) and actuator fault (4), the leader–follower tracking problem is to design distributed control protocols u_i for all the agents (1) in $\mathcal{G}$, such that the error μ_i is UUB.

3. Control protocols to address sensor faults

In this section, we consider the leader–follower tracking problem under the sensor fault only (3) (see Problem 1). This means that the actuator fault (4) satisfies $\delta_i^a = 0$ such that $\bar{u}_i = u_i$ holds for all the agents (1). Two control methods, an adaptive compensation protocol and an H_∞ control protocol, are given to handle the sensor fault problem. It will be shown that fault model assumption required in H_∞ control (Assumption 3) is weaker than the one required in adaptive control (Assumption 2).

3.1. Adaptive control design for sensor fault compensation

In this subsection, we propose an adaptive disturbance compensator to deal with sensor faults. This protocol is given as

$$u_i = -K\hat{x}_i, \tag{6}$$

$$\dot{\hat{x}}_i = F(\tilde{\bar{x}}_i - r_i) + (A - BK)\hat{x}_i - \hat{d}_i^s, \tag{7}$$

$$\dot{r}_i = Ar_i + cF_o(\sum_{j\in\mathcal{N}_i} a_{ij}(r_j - r_i) + g_i(x_0 - r_i)), \tag{8}$$

$$\dot{\hat{d}}_i^s = P_{s4}^T\hat{x}_i + P_{s2}^T(\bar{x}_i - r_i) - a_s\hat{d}_i^s, \tag{9}$$

where $\tilde{\bar{x}}_i = \bar{x}_i - \hat{x}_i$; c and a_s are positive constants; r_i denotes a distributed leader observer; controller gain K, observer gains F, F_o, and compensator gains P_{s2}, P_{s4} are to be determined later; and $\hat{d}_i^s = [\hat{d}_{i1}^{s\,T}, \hat{d}_{i2}^{s\,T}, \ldots, \hat{d}_{in}^{s\,T}]^T$ denotes a local sensor fault compensator to estimate $d_i^s = [d_{i1}^{s\,T}, d_{i2}^{s\,T}, \ldots, d_{in}^{s\,T}]^T$ with $d_i^s = F\delta_i^s$.

Remark 1. Here, we stress that it has been a difficult problem to handle sensor faults by using adaptive control techniques. Adaptive control works based on the assumption that the error variables in the closed-loop system should be accessible for the control design. However, when the sensor fault δ_i^s is involved, the actual state x_i is corrupted as shown in (3). Only the corrupted $\bar{x}_i$ can be measured. Hence, adaptive control cannot be applied in a straightforward manner. To address this issue, we propose the adaptive compensation protocol that contains two local adaptive laws (7) and (9). As seen in Theorem 1, this protocol solves Problem 1.

Assumption 2. The sensor fault δ_i^s in (3) is bounded, and its derivative $\dot{\delta}_i^s$ is bounded.

It should be noted that Assumption 2 covers various types of faults in practice. It is a weak assumption compared to those in the existing literature (Arabi et al., 2016), where not only the fault is assumed bounded, but also its upper and lower bounds must be known.

Define a global observer gain matrix as

$$A_0 = I_N \otimes A - c(L+G)\otimes F_o. \tag{10}$$

Let the design matrices $Q = Q^T \in \mathbb{R}^{n\times n}$, $R = R^T \in \mathbb{R}^{m\times m}$, $Q_o = Q_o^T \in \mathbb{R}^{n\times n}$, and $R_o = R_o^T \in \mathbb{R}^{p\times p}$ be positive-definite. Let $\sigma_i = r_i - x_0$. Design the controller gain K as

$$K = R^{-1}B^TP, \tag{11}$$

where $P > 0$ is the unique solution of the control algebraic Riccati equation (ARE)

$$0 = A^TP + PA + Q - PBR^{-1}B^TP. \tag{12}$$

Let the observer gains F and F_o be designed as

$$F = F_o = P_oR_o^{-1}, \tag{13}$$

where $P_o > 0$ is the unique solution of the observer ARE $= AP_o + P_oA^T + Q_o - P_oR_o^{-1}P_o$. Select

$$c \ge \frac{1}{2\min_{i\in\mathcal{V}}\{Re(\lambda_i(L+G))\}}. \tag{14}$$

The following is our main result in this subsection.

Theorem 1. *Suppose that the graph $\mathcal{G}$ and the sensor fault δ_i^s in (3) satisfy Assumptions 1 and 2, respectively. Let the controller design follow (11)–(14). Then, Problem 1 for the leader–follower tracking is solved by the adaptive protocol (6)–(9). Moreover, $\lim_{t\to\infty}\sigma_i(t) = 0$ at the rate of exponential convergence, and $(d_i^s - \hat{d}_i^s)$ is UUB.* □

Proof. Considering N agents in the graph $\mathcal{G}$, we define the following variables to facilitate the analysis. Let $\mu = [\mu_1^T, \mu_2^T, \ldots, \mu_N^T]^T$, $\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_N^T]^T$, $\theta = [\mu^T, \eta^T]^T$, $\sigma = [\sigma_1^T, \sigma_2^T, \ldots, \sigma_N^T]^T$, $r = [r_1^T, r_2^T, \ldots, r_N^T]^T$, and $\rho = [\rho_1^T, \rho_2^T, \ldots, \rho_N^T]^T$, where $\eta_i = \hat{x}_i$ and $\rho_i = [\mu_i^T, \eta_i^T]^T$. Hence, $\dot{\mu} = (I_N \otimes A)\mu - (I_N \otimes BK)\eta$ and $\dot{\eta} = (I_N \otimes (A - BK - F))\eta + (I_N \otimes F)\mu + (I_N \otimes F)\delta^s - (I_N \otimes F)\sigma - \hat{d}^s$. We have

$$\dot{\theta} = A_s\theta - \begin{bmatrix} 0 \\ (I_N \otimes F)\sigma \end{bmatrix} + \begin{bmatrix} 0 \\ d^s - \hat{d}^s \end{bmatrix}, \tag{15}$$

where $A_s = [as_{ij}]$ with $as_{11} = I_N \otimes A$, $as_{12} = -I_N \otimes BK$, $as_{21} = I_N \otimes F$, and $as_{22} = I_N \otimes (A - BK - F)$. Therefore, from Zhang et al. (2011), if K satisfies (11) and F satisfies (13), then both $A - BK$ and $A - F$ are Hurwitz so that A_s is Hurwitz.

At this stage, we have finished the stability analysis for the first term at the right hand side of (15). Next, we focus on the stability analysis for $(I_N \otimes F)\sigma$. From (2), (8) and (10), one has $\dot{\sigma} = A_0\sigma$.

Similar to the analysis for A_s, it is equivalent to the condition that $A-c\lambda_i(L+G)F_o$ is Hurwitz. Since F_o satisfies (13) and c satisfies (14), it is shown that A_0 is Hurwitz such that $P_{A_0}A_0 + A_0{}^T P_{A_0} = -Q_{A_0}$, where $P_{A_0} > 0$ and $Q_{A_0} > 0$. Differentiating $V_{A_0} = \sigma^T P_{A_0}\sigma$ with the respect to time t yields $\dot{V}_{A_0} = -\sigma^T Q_{A_0}\sigma$. Considering that $Q_{A_0} > 0$, we have $\sigma^T Q_{A_0}\sigma \geq \delta_{\min}(Q_{A_0})\sigma^T\sigma$. Thus, one has $\dot{V}_{A_0} \leq -\alpha V_{A_0}$, where $\alpha = \frac{\delta_{\min}(Q_{A_0})}{\delta_{\max}(P_{A_0})}$ and the solution must satisfy $\|\sigma\|^2 \leq \frac{V_{A_0}(0)}{\delta_{\min}(P_{A_0})}e^{-\alpha t}$, which implies that $r_i - x_0$ exponentially decays to zero. In what follows, we will show overall system stability analysis of (15). Because of the previous analysis of A_s, given any matrix $Q_s > 0$, there exists $P_s = [ps_{ij}] > 0$ with $ps_{11} = P_{s1}$, $ps_{12} = P_{s2}$, $ps_{21} = P_{s3}$, and $ps_{22} = P_{s4}$, such that $P_s A_s^0 + A_s^{0\,T} P_s = -Q_s$, where $A_s^0 = [as_{ij}^0]$ with $as_{11}^0 = A$, $as_{12}^0 = -BK$, $as_{21}^0 = F$, and $as_{22}^0 = A - BK - F$ is defined as a part of A_s. Define a Lyapunov function candidate as $V = \sum_{i=1}^N \rho_i^T P_s\rho_i + \tilde{d}^{s\,T}\tilde{d}^s$, where $\tilde{d}^s = d^s - \hat{d}^s$. Differentiating V with the respect to time t yields

$$\dot{V} = -\sum_{i=1}^N \rho_i^T Q_s\rho_i - 2\sum_{i=1}^N \tilde{d}_i^{s\,T} P_{s2}^T\delta_i^s + 2\sum_{i=1}^N \tilde{d}_i^{s\,T} P_{s2}^T\sigma_i - 2\sum_{i=1}^N \rho_i^T P_s\begin{bmatrix} 0 \\ F\sigma_i \end{bmatrix} + 2a_s\tilde{d}^{s\,T}\hat{d}^s + 2\tilde{d}^{s\,T}\dot{d}^s, \tag{16}$$

where (9) and (15) are employed. Considering $2a_s\tilde{d}^{s\,T}\hat{d}^s \leq -a_s\tilde{d}^{s\,T}\tilde{d}^s + a_s d^{s\,T}d^s$, $-2\tilde{d}_i^{s\,T}P_{s2}^T\delta_i^s \leq \frac{a_s}{4N}\tilde{d}_i^{s\,T}\tilde{d}_i^s + \frac{4N}{a_s}(P_{s2}^T\delta_i^s)^T P_{s2}^T\delta_i^s$, and $2\tilde{d}^{s\,T}\dot{d}^s \leq \frac{1}{4}a_s\tilde{d}^{s\,T}\tilde{d}^s + 4\frac{1}{a_s}\dot{d}^{s\,T}\dot{d}^s$, one changes (16) as

$$\dot{V} \leq -\sum_{i=1}^N \rho_i^T(Q_s - \|P_s\|^2\|F\|^2\|\sigma_i\|^2)\rho_i - (\frac{1}{2}a_s - \|P_{s2}\|^2\|\sigma_i\|^2)\tilde{d}^{s\,T}\tilde{d}^s + b_0, \tag{17}$$

where $b_0 = 2N + \sum_{i=1}^N \frac{4N}{a_s}\|P_{s2}^T\delta_i^s\|^2 + \frac{1}{a_s}d^{s\,T}d^s + \frac{4}{a_s}\dot{d}^{s\,T}\dot{d}^s$.

To finish the remaining analysis, we define $\beta_1 = \|P_s\|^2\|F\|^2\frac{V_{A_0}(0)}{\delta_{\min}(P_{A_0})}$ and $\beta_2 = \|P_{s2}\|^2\frac{V_{A_0}(0)}{\delta_{\min}(P_{A_0})}$. Note that β_1, β_2, and b_0 are all bounded. Let a constant $\bar{b}_0$ be the upper bound of b_0 and a constant ϑ satisfy $0 < \vartheta < \delta_{\min}(Q_s)$. With above definitions, (17) yields

$$\dot{V} \leq -\sum_{i=1}^N \rho_i^T(\delta_{\min}(Q_s) - \beta_1 e^{-\alpha t})\rho_i - (\frac{1}{2}a_s - \beta_2 e^{-\alpha t})\tilde{d}^{s\,T}\tilde{d}^s + \bar{b}_0. \tag{18}$$

In addition, there exists $T_1 > 0$ such that for all $t \geq T_1$, one has $q_1 \equiv \delta_{\min}(Q_s) - \vartheta_1 \leq \delta_{\min}(Q_s) - \beta_1 e^{-\alpha t}$. Similarly, there exists $T_2 > 0$ such that for all $t \geq T_2$, one has $q_2 \equiv \frac{1}{2}a_s - \vartheta_2 \leq \frac{1}{2}a_s - \beta_2 e^{-\alpha t}$. Thus, (18) is changed to $\dot{V}(t) \leq -q_1\rho^T\rho - q_2\tilde{d}^{s\,T}\tilde{d}^s + \bar{b}_0$, $t \geq t_1$, where $t_1 = \max\{T_1, T_2\}$. Now, integrate $\dot{V}(t)$ with time to yield

$$V(t) \leq (V(t_1) - \frac{\bar{b}_0}{a_0})e^{-a_0 t} + \frac{\bar{b}_0}{a_0}, \quad t \geq t_1, \tag{19}$$

where $a_0 = \min\{\frac{q_1}{\delta_{\max}(P_s)}, q_2\}$. Note that a_0 is a positive constant. From (19), it reveals that all the signals in $V(t)$ including μ are UUB. Moreover, from (19), we have $\|\mu(t)\|^2 \leq 2(V(t_1) - \frac{\bar{b}_0}{a_0})e^{-a_0 t} + 2\frac{\bar{b}_0}{a_0}$ for $t \geq t_1$, which implies that $\lim_{t\to\infty}\|\mu(t)\|^2 \leq 2\frac{\bar{b}_0}{a_0}$. Hence, the proof is completed. ■

Note that the matrix A_s in (15) is fully decoupled, and its design gains are locally determined by each agent. This is different from the corresponding matrix A_θ in Section 5.C of Zhang et al. (2011), whose protocol is designed based on the distributed information. The proposed decoupled matrix A_s helps construct the adaptive control mechanism and compensate the sensor fault. Compared to Zhang et al. (2011), extra terms $(I_N \otimes F)\sigma$ and $d^s - \hat{d}^s$ are generated as shown in (15).

3.2. H_∞ Control Design for Sensor Fault Compensation

In the previous subsection, the leader–follower tracking was achieved under Assumption 2, where both sensor fault δ_i^s in (3) and its derivative $\dot{\delta}_i^s$ are assumed bounded. In general, the sensor faults may be added based on the agents' states, and thus the boundedness of sensor faults cannot be ensured. As a result, the protocol in the previous subsection may not work in some applications. To remove the bounded Assumption 2 on sensor faults, an H_∞ control scheme is designed in this subsection.

Before the control design, we give the following definition and lemma about static output-feedback control design. More details can be found in Gadewadikar, Lewis, and Abu-Khalaf (2006).

Definition 2. Define a linear time-invariant system as $\dot{x} = \bar{A}x + \bar{B}u + \bar{D}d$, $y = \bar{C}x$, where u, y, and d denote the system input, output, and disturbance, respectively. Define a performance output w as $\|w\|^2 = x^T\bar{Q}x + u^T\bar{R}u$ for $\bar{Q} \geq 0$ and $\bar{R} > 0$. The system L_2 gain is said to be bounded or attenuated by γ if the L_2 norms of w and d satisfy: $\frac{\int_0^\infty \|w\|^2 dt}{\int_0^\infty \|d\|^2 dt} = \frac{\int_0^\infty (x^T\bar{Q}x + u^T\bar{R}u)dt}{\int_0^\infty (d^T d)dt} \leq \gamma^2$.

Lemma 1 (*Gadewadikar et al., 2006*). *Assume that $(\bar{A}, \sqrt{\bar{Q}})$ is detectable with $\bar{Q} \geq 0$. Then, the system considered in Definition 2 is output-feedback stabilizable with L_2 gain bounded by γ, if and only if (1) there exist matrices $\bar{K}$, M, and $\bar{P}$ such that*

$$\bar{K}\bar{C} = \bar{R}^{-1}(\bar{B}^T\bar{P} + M), \tag{20}$$

$$\bar{P}\bar{A} + \bar{A}^T\bar{P} + \bar{Q} + \gamma^{-2}\bar{P}\bar{D}\bar{D}^T\bar{P} + M^T\bar{R}^{-1}M = \bar{P}\bar{B}\bar{R}^{-1}\bar{B}^T\bar{P}, \tag{21}$$

and (2) $(\bar{A}, \bar{B})$ is stabilizable and $(\bar{A}, \bar{C})$ is detectable. □

A key quantity in rejecting sensor faults is the error

$$e_i = \bar{x}_i - \hat{x}_i - \hat{\delta}_i^s, \tag{22}$$

where $\hat{x}_i$ is an estimate of the uncorrupted state x_i, and $\hat{\delta}_i^s$ is an estimate of the sensor fault δ_i^s. Note that e_i can be measured. Propose now an H_∞ control protocol to reject sensor faults as

$$u_i = cK\hat{\epsilon}_i, \tag{23}$$

$$\hat{\epsilon}_i = \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) + g_i(x_0 - \hat{x}_i), \tag{24}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + cBK\hat{\epsilon}_i + w_i, \tag{25}$$

$$w_i = (F_1 + F_2)e_i, \tag{26}$$

$$\dot{\hat{\delta}}_i^s = -F_1 e_i, \tag{27}$$

where the controller gain K and observer gains F_1 and F_2 are determined later. Let the estimation error for the sensor fault be $\tilde{\delta}_i^s = \delta_i^s - \hat{\delta}_i^s$.

Assumption 3. The derivative $\dot{\delta}_i^s$ of the sensor fault in (3) is bounded.

Define a global controller gain matrix as

$$A_c = I_N \otimes A - c(L + G) \otimes BK. \tag{28}$$

The following is our main result in this subsection.

Theorem 2. *Suppose that the graph $\mathcal{G}$ satisfies Assumption 1, the sensor fault satisfies Assumption 3, and A is nonsingular. Let K and c be designed as (11) and (14). Suppose F_1 and F_2 are chosen such that $\bar{K} = [F_2^T, -F_1^T]^T$ follows (20) and (21) in Lemma 1. Then, Problem 1 for the leader–follower tracking under the sensor fault (3) is solved by the H_∞ control protocol (23)–(27). Moreover, the L_2 gains of the errors e_i and $\tilde{\delta}_i^s$ are bounded in terms of the L_2 norms of disturbances $d_i = [\dot{\delta}_i^{s^T}, \dot{\delta}_i^{s^T}]^T$.* □

Proof. Define $d = [d_1^T, d_2^T, \ldots, d_N^T]^T$, $\hat{\zeta}_i = \hat{x}_i - x_0$, $e = [e_1^T, e_2^T, \ldots, e_N^T]^T$, $\hat{\zeta} = [\hat{\zeta}_1^T, \hat{\zeta}_2^T, \ldots, \hat{\zeta}_N^T]^T$, and $\tilde{\delta}^s = [\tilde{\delta}_1^{s^T}, \tilde{\delta}_2^{s^T}, \ldots, \tilde{\delta}_N^{s^T}]^T$. From (1), (25), and (27), differentiating e_i in (22) with respect to time t yields $\dot{e}_i = (A - F_2)e_i - A\tilde{\delta}_i^s + \dot{\delta}_i^s$. From (27), it is thus shown that

$$[\dot{e}_i, \dot{\tilde{\delta}}_i^s]^T = A_{F1}[e_i, \tilde{\delta}_i^s]^T + d_i. \quad (29)$$

where $A_{F1} = [af_{ij}^1]$ with $af_{11}^1 = A - F_2$, $af_{12}^1 = -A$, $af_{21}^1 = F_1$, and $af_{22}^1 = 0$ and $d_i = [\dot{\delta}_i^s, \dot{\delta}_i^s]^T$. In the following, we will show the stabilization of (29) can be achieved by appropriately designing F_1 and F_2. To do this, we transform (29) to the following output-feedback control system

$$\dot{x}_T \triangleq \bar{A}x_T + \bar{B}u_T + \bar{D}d_i, \quad y_T \triangleq \bar{C}x_T, \quad (30)$$

where $\bar{A} = [A, -A; 0, 0]$, $\bar{B} = \bar{D} = [I_N, 0; 0, I_N]$, and $\bar{C} = [I_N, 0]$. Moreover, define the controller u_T (30) as $u_T = -\bar{K}y_T$, where $\bar{K} = [K_1^T, K_2^T]^T$. Straightforward analysis shows that if we choose $K_1 = F_2$ and $K_2 = -F_1$, then (30) is equivalent to (29). At this stage, we focus on finding appropriate design matrices for $\bar{K}$ by using the static output-feedback control technique. From the structures of $\bar{A}$ and $\bar{B}$, it is obtained that the pair $(\bar{A}, \bar{B})$ is stabilizable. Note that the pair $(\bar{A}, \bar{C})$ is detectable since A is nonsingular. Moreover, we select $\bar{Q} = I_{2n}$ such that $(\bar{A}, \sqrt{\bar{Q}})$ is detectable. From Lemma 1, if one has matrices $\bar{K}$, M and $\bar{P}$ satisfy (20) and (21), then the system described by (30) is guaranteed output-feedback stabilizable. As a result, $A_{F1} = \bar{A} - \bar{B}\bar{K}\bar{C}$ is Hurwitz, and the L_2 gain of (30) is bounded in terms of the L_2 norm of d_i.

With A_{F1} Hurwitz, given any matrix $Q_{F1} > 0$, there exists a matrix $P_{F1} > 0$ such that $P_{F1}^T A_{F1} + A_{F1}P_{F1} = -Q_{F1}$. In order to analyze (29), we choose $V_{F1} = \theta_F^T(I_N \otimes P_{F1})\theta_F$, where $\theta_F = [e^T, \tilde{\delta}^{s^T}]^T$. Taking the derivative of V_{F1} yields $\dot{V}_{F1} = -\theta_F^T(I_N \otimes Q_{F1})\theta_F + 2\theta_F^T(I_N \otimes P_{F1})d \leq -\delta_{\min}(Q_{F1})\|I_{Nn}\theta_F\|^2 + 2\|\theta_F\|\|I_N \otimes P_{F1}\|\|d\|$, where the boundedness of θ_F is obtained.

To finish the proof, we will show the leader–follower tracking under the proposed control. From (25), the derivative of $\hat{\zeta}_i$ yields $\dot{\hat{\zeta}} = (I_N \otimes (F_1 + F_2))e + A_c\hat{\zeta}$. Selecting K and c as (11) and (14), one has that A_c in (28) is Hurwitz. This means that given any matrix $Q_c > 0$, there exists a matrix $P_c > 0$ such that $P_c^T A_c + A_c P_c = -Q_c$. Let $V_{\hat{\zeta}} = \hat{\zeta}^T P_c \hat{\zeta}$. Taking the derivative of $V_{\hat{\zeta}}$ with respect to time t yields $\dot{V}_{\hat{\zeta}} = -\hat{\zeta}^T Q_c \hat{\zeta} + 2\hat{\zeta}^T(I_N \otimes (F_1 + F_2))e$, where e is bounded for any time because of the boundedness of θ_F. This leads that $\hat{\zeta}_i$ is bounded. Then, the proposed H_∞ control protocol ensures that all the agents converge to the neighborhood of the leader in the presence of sensor faults. This completes the proof. ■

Remark 2. The design protocols of Theorems 1 and 2 are in some sense dual. Theorem 1 relies on a local design for the feedback gain K, and a global design for the observer gain F_0 in A_0 of (10). On the other hand, Theorem 2 relies on a local design for the observer gains F_1 and F_2, and a global design for the controller gain K in A_c of (28).

4. Control protocols to address sensor and actuator faults

In this section, we consider a more general case, where both the sensor fault (3) and actuator fault (4) are involved. We propose two control methods, including an adaptive compensation control scheme and an H_∞ control scheme.

4.1. Adaptive control design for sensor and actuator fault Compensation

In this subsection, we introduce an adaptive compensation scheme to handle sensor and actuator faults. To facilitate the analysis, we make the following assumption.

Assumption 4. The actuator fault δ_i^a in (4) and its derivative $\dot{\delta}_i^a$, are bounded. Moreover, the sensor fault δ_i^s in (3) and its derivative $\dot{\delta}_i^s$, are bounded.

The control scheme is given as follows

$$u_i = cK\hat{\epsilon}_i, \quad (31)$$

$$\hat{\epsilon}_i = \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) + g_i(x_0 - \hat{x}_i), \quad (32)$$

$$\dot{\hat{x}}_i = A\hat{x}_i + cBK\hat{\epsilon}_i + w_i + \hat{d}_i^a, \quad (33)$$

$$w_i = (F_1 + F_2)e_i, \quad (34)$$

$$\dot{\hat{\delta}}_i^s = -F_1 e_i, \quad (35)$$

$$\dot{\hat{d}}_i^a = P_{F21}^T e_i - P_{F23}^T \tilde{\delta}_i^s - a_a \hat{d}_i^a, \quad (36)$$

where $\hat{x}_i$ denotes a distributed uncorrupted state observer; a_a is a positive constant; control gains K, c, observer gains F_1, F_2, and compensator gains P_{F21}, P_{F23} are designed later; and $\hat{d}_i^a$ denotes an actuator fault compensator to estimate d_i^a with $d_i^a = B\delta_i^a$.

Theorem 3. *Suppose that the graph $\mathcal{G}$ satisfies Assumption 1, the sensor and actuator faults satisfy Assumption 4, and A in (1) is nonsingular. Design K as (11), F_1 and F_2 as $\bar{K} = [F_1^T, -F_2^T]^T$ satisfying Lemma 1 with $\bar{A}$, $\bar{B}$, $\bar{C}$ given in (30). Then, Problem 2 for the leader–follower tracking under the sensor fault (3) and actuator fault (4) is solved by the adaptive protocol (31)–(36). Moreover, $(d_i^a - \hat{d}_i^a)$ is UUB.* □

Proof. Differentiating the error e_i (22) with the respect to time t yields $\dot{e}_i = (A - F_2)e_i - A\tilde{\delta}_i - \hat{d}_i^a + B\delta_i^a + \dot{\delta}_i^s$, where (3), (4), (33), and (35) are employed. Thus, it is shown that $\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{\delta}}_i^s \end{bmatrix} = A_{F1}\begin{bmatrix} e_i \\ \tilde{\delta}_i^s \end{bmatrix} + \begin{bmatrix} B\delta_i^a - \hat{d}_i^a \\ 0 \end{bmatrix} + d_i$, where A_{F1} and d_i are defined in (29).

Similar to analyses in (30), we can design F_1 and F_2 to satisfy (20) and (21) such that A_{F1} is Hurwitz. Therefore, given any matrix $Q_{F2} > 0$, there exists $P_{F2}^T = P_{F2} \equiv \begin{bmatrix} P_{F21} & P_{F22} \\ P_{F23} & P_{F24} \end{bmatrix} > 0$ such that the following constraint holds $P_{F2}A_{F1} + A_{F1}^T P_{F2} = -Q_{F2}$. Define $V = \sum_{i=1}^N \varrho_i^T P_{F2}\varrho_i + \tilde{d}^{a^T}\tilde{d}^a$, where $\varrho_i = [e_i^T, \tilde{\delta}_i^{s^T}]^T$ and $\tilde{d}^a = d^a - \hat{d}^a$. Differentiating V with the respect to time t yields

$$\dot{V} = -\sum_{i=1}^N \varrho_i^T Q_{F2}\varrho_i + 2\sum_{i=1}^N \delta_i^{s^T} P_{F23}\tilde{d}_i^a + 2a_a\tilde{d}^{a^T}\hat{d}^a$$
$$+2\sum_{i=1}^N \varrho_i^T P_{F2}d_i + 2\tilde{d}^{a^T}\dot{d}^a. \quad (37)$$

The rest of the proof is similar to that of Theorem 1. Thus, the leader–follower tracking is reached despite the sensor and actuator faults, and Problem 2 is solved. ■

4.2. H_∞ Control Design for Sensor and Actuator Fault Compensation

To remove the requirements on sensor and actuator faults in Assumption 4, we propose an H_∞ control protocol in this subsection.

The H_∞ control scheme to reject sensor and actuator faults is given as follows

$$u_i = cK\hat{\epsilon}_i, \tag{38}$$

$$\hat{\epsilon}_i = \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_j - \hat{x}_i) + g_i(x_0 - \hat{x}_i), \tag{39}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + cBK\hat{\epsilon}_i + w_i, \tag{40}$$

$$w_i = (F_1 + F_2)e_i, \tag{41}$$

$$\dot{\hat{\delta}}^s_i = -F_1 e_i, \tag{42}$$

where $\hat{x}_i$ is an uncorrupted state observer.

Assumption 5. The derivative $\dot{\delta}^s_i$ of the sensor fault in (3) is bounded. The actuator fault δ^a_i in (4) is bounded.

Theorem 4. *Suppose that the graph $\mathcal{G}$ satisfies Assumption 1, the sensor and actuator faults satisfy Assumption 5, and A in (1) is nonsingular. Design K as (11) and (12), and F_1 and F_2 as $\bar{K} = [F_2^T, -F_1^T]^T$ satisfying (20) and (21). Then, Problem 2 for the leader–follower tracking under the sensor fault (3) and actuator fault (4) is solved by an H_∞ control protocol (38)–(42). Moreover, the L_2 gains of the errors e_i and $\tilde{\delta}^s_i$ are bounded in terms of the L_2 norms of disturbances $d_i = [\dot{\delta}^{sT}_i, (B\delta^a_i)^T]^T$.* □

Proof. The stability of the overall system is decided by two decoupled subsystems, namely, $[\dot{e}^T, \dot{\tilde{\delta}}^{sT}]^T = \begin{bmatrix} I_N\otimes(A-F_2) & -I_N\otimes A \\ I_N\otimes F_1 & 0\end{bmatrix}\begin{bmatrix} e \\ \tilde{\delta}^s\end{bmatrix} + \begin{bmatrix}\dot{\delta}^s + (I_N\otimes B)\delta^a \\ \dot{\delta}^s\end{bmatrix}$, and $\dot{\hat{\zeta}} = (I_N\otimes(F_1+F_2))e + A_c\hat{\zeta}$. The rest of the proof is similar to that of Theorem 2. Thus, the leader–follower tracking is reached in the presence of sensor and actuator faults by the H_∞ control protocol (38)–(42). ■

Remark 3. The proposed control protocols in Theorems 3 and 4 address simultaneous sensor and actuator faults in a unified platform, which is a generalized case of the results in Theorems 1 and 2. Moreover, Theorems 1 and 3 mainly use the adaptive control technique to handle faults, while Theorems 2 and 4 use the H_∞ control technique. These two different control techniques require complementary assumptions on the system modeling, including the fault model and system dynamics. Specifically, in our adaptive control based protocols, the fault and its derivative are required to be bounded. The requirement of a bounded sensor fault is relaxed in our H_∞ control protocols at the expense of limiting the system dynamics A to be nonsingular. The complementarities between adaptive control and H_∞ control can be further found in their different control design philosophies. Adaptive controls concern *online* adaptive law design, while H_∞ controls concern *off-line* matrix design.

5. Simulation studies

In this section, we present simulations to demonstrate the resilience of the proposed protocols against faults on sensors and actuators. Note that a variety of phenomena in industrial systems can be described by a mass–spring system, such as the movement of deformable objects and vibration in mechanical devices (Lewis et al., 2013). Hence, we use a group of mass–spring systems as a test bed. Specifically, the simulated MAS has four followers and a leader subject to a directed graph, $\mathcal{G}$, given in Fig. 1. Each follower in the graph is a two-mass–spring system, modeled as (1), where $A = [a_{ij}]$ with $a_{12} = a_{34} = 1$, $a_{21} = \frac{-(k_1+k_2)}{m_1}$, $a_{23} = -a_{41} = -a_{43} = \frac{k_2}{m_1}$ and otherwise zeros and $B = [0,\ \frac{1}{m_1},\ 0,\ 0]^T$. Here, m_1 and m_2 are masses, k_1 and k_2 are spring constants, u_i is the input applied on mass 1, and $x_i = [y_{i,1}, \dot{y}_{i,1}, y_{i,2}, \dot{y}_{i,2}]^T$ with $y_{i,1}$ and $y_{i,2}$ being the displacements of the two masses and $\dot{y}_{i,1}$ and $\dot{y}_{i,2}$ being the velocities. The leader is unforced and has the same parameters as the follower does. For the simulation, we choose $k_1 = 3N/m$, $k_2 = 2N/m$, $m_1 = 1.1$ kg, and $m_2 = 0.9$ kg. The simulation objective to make displacements of the two masses, $y_{i,1}$ and $y_{i,2}$, synchronize to that of the leader $y_{0,1}$ and $y_{0,2}$, respectively, i.e. $\lim_{t\to\infty}(y_{i,1} - y_{0,1}) = 0$ and $\lim_{t\to\infty}(y_{i,2} - y_{0,2}) = 0$.

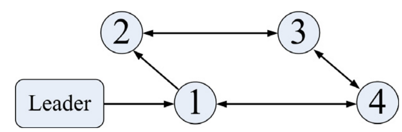

Fig. 1. Graph $\mathcal{G}$ used for the leader–follower tracking.

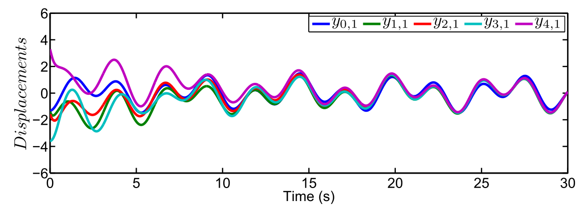

Fig. 2. Leader–follower tracking performance: $y_{0,1}, y_{i,1}$.

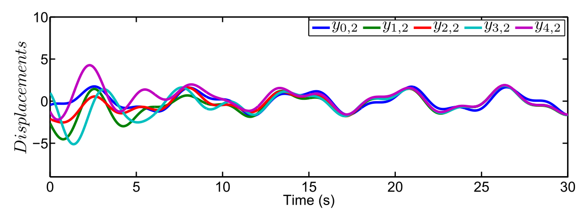

Fig. 3. Leader–follower tracking performance: $y_{0,2}, y_{i,2}$.

In what follows, we implement the control protocols by following Theorems 3 and 4. The implementation of Theorems 1 and 2 is a simplified case of Theorems 3 and 4 and thus is omitted due to the limited space. Let us first verify Theorem 3. The sensor fault δ^s_i in (3) and actuator fault δ^a_i in (4) are considered as $\delta^s_i = \delta^a_i = [0.5\sin(t), 0.5\sin(t), 0.5\sin(t), 0.5\sin(t)]^T$. It is clear that δ^s_i, $\dot{\delta}^s_i$, δ^a_i, and $\dot{\delta}^a_i$ are bounded. This satisfies the condition of the sensor and actuator faults required in Theorem 3. The controller gain in (11) is designed as $K = [0.7885, 1.6537, -0.4718, 0.7075]$ by solving control ARE (12). Select the coupling gain $c = 15$ to satisfy (14). Design Q_{F2} as an identity matrix. The initial values of updating parameters $\hat{x}(0)$, $\hat{\delta}^s_i(0)$, and $\hat{d}^a(0)$ in (33), (35), and (36) are set to zero. The initial states of the leader and followers are randomly chosen. After applying the adaptive control protocol in Theorem 3, we plot agents' trajectories in Figs. 2 and 3. It is shown that the resilience is guaranteed in the sense that all the followers converge to the leader in the presence of unknown sensor and actuator faults.

Now, we are in a position to verify our result in Theorem 4, where the bounded sensor faults in the previous simulation are now relaxed to be unbounded. To this end, we consider the sensor fault in (3) and the actuator fault in (4) as $\delta^s_i = [0.2t, 0, 0, 0]^T$ and $\delta^a_i = [0.1\sin(t), 0.1\sin(t), 0.1\sin(t), 0.1\sin(t)]^T$. The matrix A is nonsingular, which meets the design condition required in Theorem 4. The controller gains K and c in (23) are the same as the ones in the previous simulation. Moreover, the initial values of system states are randomly set, while updating parameters are initially set to be zero. The trajectories of all agents are presented in Figs. 4 and 5. It reveals that the resilient control of MAS with sensor

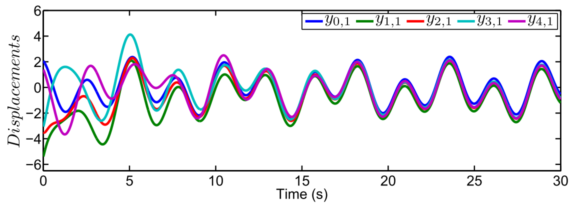

Fig. 4. Leader–follower tracking performance: $y_{0,1}, y_{i,1}$.

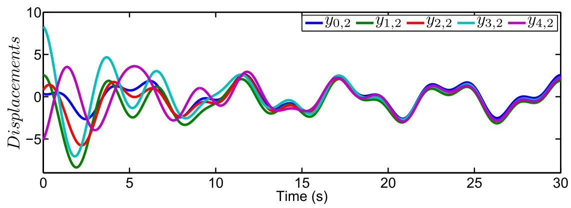

Fig. 5. Leader–follower tracking performance: $y_{0,2}, y_{i,2}$.

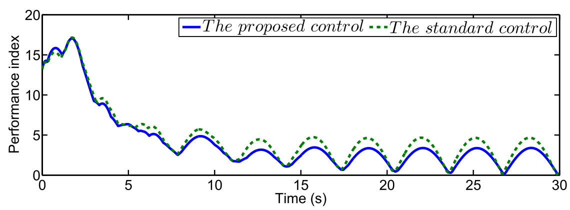

Fig. 6. Performance index.

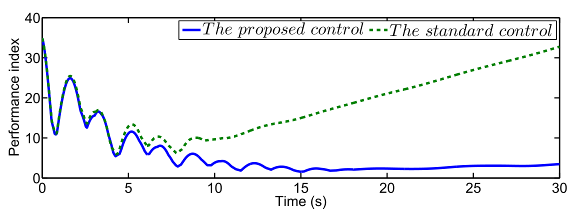

Fig. 7. Performance index.

and actuator faults is well achieved after applying the proposed H_∞ control protocol in Theorem 4.

In addition, a performance index is defined as $Index = \sum_{i=1}^{4} (|y_{i,1} - y_{0,1}| + |y_{i,2} - y_{0,2}|)$, which denotes the sum of absolute displacement errors between the leader and followers. Here, we use the results in Figs. 2–5 and calculate the corresponding performance indexes in Figs. 6 and 7. Moreover, the standard control (5) from (Zhang et al., 2011) is applied to the mass–spring system, where the system parameters including the initial system states and faults are the same as those in the previous simulations. For the comparison, the performance indexes in presence of the standard control (5) are also plotted in Figs. 6 and 7. It is clear that the proposed methods provide better resiliency for the synchronization of two-mass–spring systems, when compared to the standard control (5).

6. Conclusion

This paper investigates the resilient design problem for MAS, and achieves the leader–follower tracking in the presence of sensor and actuator faults by using adaptive compensation controls and H_∞ controls. To achieve adaptive compensation control, we provide the resilience by employing a local sensor/actuator fault compensator. Moreover, H_∞ controls are proposed by using static output-feedback design technique, which allows us to extend sensor faults to unbounded cases, and, thus, to further improve the resilience. Finally, the effectiveness of the proposed protocols has been validated by simulation studies. It is an important research topic to extend our results in the context of insecure inter-agent communications. Moreover, how to relax the assumption of the nonsingular system dynamics A is also a challenging task. We will consider such problems in the future.

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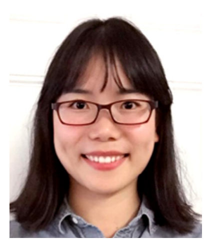

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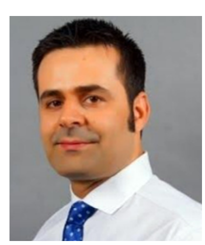

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