MECH5315M Engineering Computational Methods: Re-sit Assignment

Formalities

The report consists of a written document and an Appendix with your Matlab code(s). You will need to submit both to get full marks. The written part should be between 1 and 2 pages long using at least font 11 and 2cm margins. Please attach the print code as an Appendix – this does not count towards your 2 page limit. **You may re-use any code that has been provided to you in Minerva, but you must clearly indicate where you have done so in the comments in your code.**

What you need to program in Matlab:

1. Write a program in Matlab that solves the heat diffusion equation

$$\frac{\partial T}{\partial t}(x,t) = \frac{\partial^2 T}{\partial x^2}(x,t), \quad 0 \leq x \leq 1.0$$

with boundary conditions, $T(0,t) = T(1.0,t) = 0$, initial conditions $T(x,0) = \sin(5\pi x)$ and centred finite differences with both explicit and implicit Euler.

2. Your program should obtain solutions with different values of Δt and Δx. You should do this by varying the number of time steps from 75 to 500 and the number of finite difference nodes from 25 to 50. In each case run both Euler methods with the given time step until $t = 0.1$ and store the logarithm of the maximum of the absolute value of the final result, that is log(max(abs(ø))), where ø is the vector containing all approximate point values T_i from your finite difference formulation.

3. Your program should use a mesh plot in Matlab to visualise how this value changes depending on Δx and Δt for implicit and explicit Euler. Your x-axis should show the width of your finite difference mesh Δx and your y-axis the time step Δt. Your program should generate this figure for both Euler methods without requiring additional input.

What you need to write in the report. Your report must be structured as described below. Only answers that are within the correct section will be awarded marks.

<u>Section 1: Introduction (20 marks in total)</u>

a) Description of the problem: introduce the problem that you are solving (5 marks).
b) Description of numerical methods: introduce the numerical methods in your code. In particular, write down in detail how you incorporate the boundary condition in your finite difference formulation. Include a list of all the problem parameters: give all the information that is required to reproduce your results. This can be in the form of a table. (15 marks)

<u>Section 2: Results (20 marks in total)</u>

a) Figures (10 marks): show the two figures emerging from your code.
b) Description of results (10 marks): describe what first can be observed in each figure individually and then what noticeable differences you can see between them. Try to be as quantitative as possible: for example, if you observe something for specific values of Δt and Δx but not for others, make sure to include the values in your description.

<u>Appendix: Matlab program (60 marks in total)</u>