

1. A policeman (not moving) uses a radar gun with a frequency of 34,700,000,000Hz to detect the speed of other vehicles. The radar is not fully functioning and only shows that the frequency of reflected signal from a detected vehicle is 34, XXX, XXX, 000 (X means missing values). If the speed limit is 35 mph, please determine if the detected vehicle violates the speed limit.

Hint: $f_o = \frac{v+v_o}{v+v_s} f_s$, where f_o =observed frequency, f_s = actual frequency of wave from the radar, the speed of sound waves $v = 760$ mph, v_o = the speed of the radar gun.

2. An automated emergency vehicle cruises on a highway segment with a length of L. The vehicle travels to a spot on the highway whenever an accident happens. Accidents happen in the highway segment uniformly. What's the expected distance that the automated vehicle will travel?
3. There is a stream of vehicles on a single lane highway. The probability of a vehicle being a human-driven vehicle (HV) is 0.4. The probability of a vehicle being an automated vehicle (AV) is 0.6. Given a vehicle is an HV, the probability of an AV following an HV is 0.9. Find the probability of an AV following a HV given a vehicle is an AV.
4. Assume that the instantaneous fuel consumption of an AV is proportional to the square of its acceleration. At time 0, the velocity and location of an AV are v^- and x^- , respectively. The AV wants to reach location x^+ at final speed v^+ in a time period $[0, T]$. The maximum velocity is \bar{v} and the acceleration fall within $[\underline{a}, \bar{a}]$. Find the most fuel-efficient trajectory for this AV.
 - (1) Mathematically formulate the problem using a continuous time variable.
 - (2) Mathematically formulate the problem using a discrete time variable, i.e., the time period is discretized into time intervals with equal length and the decision becomes finding the trajectory of the vehicle during each time interval.
 - (3) Choose a programming language that you prefer, create a small instance of the problem and solve the problem using the models in (1) and (2).